

5-1

Midsegments of Triangles

Objective To use properties of midsegments to solve problems



You know about midpoints, and you know about segments. This problem combines the two.



Getting Ready!

Cut out a triangle of any shape. Label its largest angle C , and the other angles A and B . Fold A onto C to find the midpoint of $\overline{AC}$. Do the same for $\overline{BC}$. Label the midpoints L and N , and then draw $\overline{LN}$.

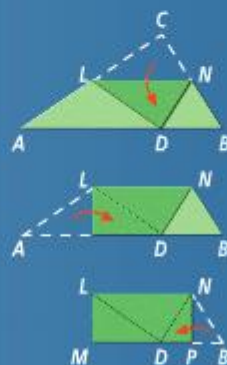
Fold the triangle on $\overline{LN}$ as shown.

Fold A to D and fold B to D .

Label the vertices M and P as shown.

What is the relationship between $\overline{MP}$ and $\overline{AB}$? How do you know?

What conjecture can you make about the relationship between $\overline{LN}$ and $\overline{AB}$?



Lesson Vocabulary

- midsegment of a triangle

In the Solve It, $\overline{LN}$ is a midsegment of $\triangle ABC$. A **midsegment of a triangle** is a segment connecting the midpoints of two sides of the triangle.

Essential Understanding There are two special relationships between a midsegment of a triangle and the third side of the triangle.



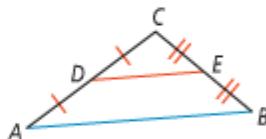
Theorem 5-1 Triangle Midsegment Theorem

Theorem

If a segment joins the midpoints of two sides of a triangle, then the segment is parallel to the third side and is half as long.

If ...

D is the midpoint of $\overline{CA}$ and
 E is the midpoint of $\overline{CB}$

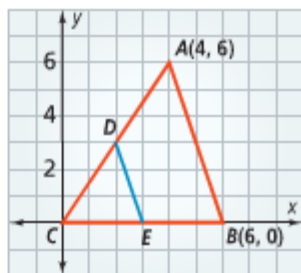


Then ...

$\overline{DE} \parallel \overline{AB}$ and
 $DE = \frac{1}{2}AB$

You will prove Theorem 5-1 in Lesson 6-9.

Here's Why It Works You can verify that the Triangle Midsegment Theorem works for a particular triangle. Use the following steps to show that $\overline{DE} \parallel \overline{AB}$ and that $DE = \frac{1}{2}AB$ for a triangle with vertices at $A(4, 6)$, $B(6, 0)$, and $C(0, 0)$, where D and E are the midpoints of $\overline{CA}$ and $\overline{CB}$.



Step 1 Use the Midpoint Formula, $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$, to find the coordinates of D and E .

The midpoint of $\overline{CA}$ is $D\left(\frac{0+4}{2}, \frac{0+6}{2}\right) = D(2, 3)$.

The midpoint of $\overline{CB}$ is $E\left(\frac{0+6}{2}, \frac{0+0}{2}\right) = E(3, 0)$.

Step 2 To show that the midsegment $\overline{DE}$ is parallel to the side $\overline{AB}$, find the slope, $m = \frac{y_2 - y_1}{x_2 - x_1}$, of each segment.

$$\begin{aligned} \text{slope of } \overline{DE} &= \frac{0-3}{3-2} & \text{slope of } \overline{AB} &= \frac{0-6}{6-4} \\ &= \frac{-3}{1} & &= \frac{-6}{2} \\ &= -3 & &= -3 \end{aligned}$$

Step 3 To show $DE = \frac{1}{2}AB$, use the Distance Formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to find DE and AB .

$$\begin{aligned} DE &= \sqrt{(3-2)^2 + (0-3)^2} & AB &= \sqrt{(6-4)^2 + (0-6)^2} \\ &= \sqrt{1+9} & &= \sqrt{4+36} \\ &= \sqrt{10} & &= \sqrt{40} \\ & & &= 2\sqrt{10} \end{aligned}$$

Since $\sqrt{10} = \frac{1}{2}(2\sqrt{10})$, you know that $DE = \frac{1}{2}AB$.

Think

How do you identify a midsegment?

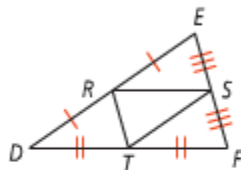
Look for indications that the endpoints of a segment are the midpoints of a side of the triangle.



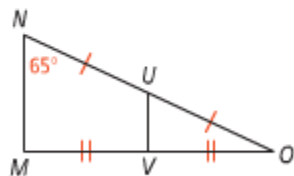
Problem 1 Identifying Parallel Segments

What are the three pairs of parallel segments in $\triangle DEF$?

$\overline{RS}$, $\overline{ST}$, and $\overline{TR}$ are the midsegments of $\triangle DEF$. By the Triangle Midsegment Theorem, $\overline{RS} \parallel \overline{DF}$, $\overline{ST} \parallel \overline{ED}$, and $\overline{TR} \parallel \overline{FE}$.



- Got It?** 1. a. In $\triangle XYZ$, A is the midpoint of $\overline{XY}$, B is the midpoint of $\overline{YZ}$, and C is the midpoint of $\overline{ZX}$. What are the three pairs of parallel segments?
b. **Reasoning** What is $m\angle VUO$ in the figure at the right? Explain your reasoning.



Plan

Which relationship stated in the Triangle Midsegment Theorem should you use?

You are asked to find lengths, so use the relationship that refers to the lengths of a midsegment and the third side.



Problem 2 Finding Lengths

In $\triangle QRS$, T , U , and B are midpoints. What are the lengths of $\overline{TU}$, $\overline{UB}$, and $\overline{QR}$?

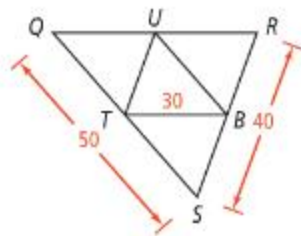
Use the relationship

length of a midsegment = $\frac{1}{2}$ (length of the third side)
to write an equation about the length of each midsegment.

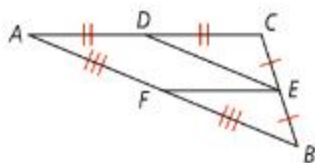
$$\begin{aligned} TU &= \frac{1}{2} SR \\ &= \frac{1}{2}(40) \\ &= 20 \end{aligned}$$

$$\begin{aligned} UB &= \frac{1}{2} QS \\ &= \frac{1}{2}(50) \\ &= 25 \end{aligned}$$

$$\begin{aligned} TB &= \frac{1}{2} QR \\ 30 &= \frac{1}{2} QR \\ 60 &= QR \end{aligned}$$



Got It? 2. In the figure at the right, $AD = 6$ and $DE = 7.5$. What are the lengths of $\overline{DC}$, $\overline{AC}$, $\overline{EF}$, and $\overline{AB}$?



You can use the Triangle Midsegment Theorem to find lengths of segments that might be difficult to measure directly.



Problem 3 Using a Midsegment of a Triangle

Environmental Science A geologist wants to determine the distance, $\overline{AB}$, across a sinkhole. Choosing a point E outside the sinkhole, she finds the distances $\overline{AE}$ and $\overline{BE}$. She locates the midpoints C and D of $\overline{AE}$ and $\overline{BE}$ and then measures $\overline{CD}$. What is the distance across the sinkhole?

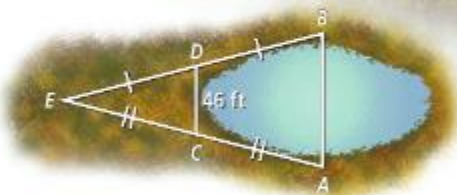
$\overline{CD}$ is a midsegment of $\triangle AEB$.

$$CD = \frac{1}{2} AB \quad \triangle \text{ Midsegment Thm.}$$

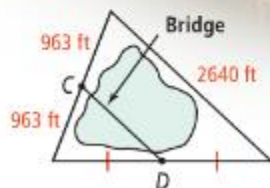
$$46 = \frac{1}{2} AB \quad \text{Substitute 46 for } CD.$$

$$92 = AB \quad \text{Multiply each side by 2.}$$

The distance across the sinkhole is 92 ft.



Got It? 3. $\overline{CD}$ is a bridge being built over a lake, as shown in the figure at the right. What is the length of the bridge?



Think

Why does the geologist find the length of $\overline{CD}$?

$\overline{CD}$ is a midsegment of $\triangle AEB$, so the geologist can use its length to find $\overline{AB}$, the distance across the sinkhole.

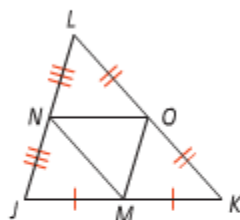


Lesson Check

Do you know HOW?

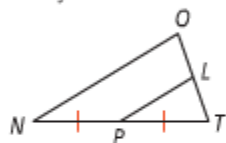
Use the figure at the right for Exercises 1–3.

- Which segment is parallel to $\overline{JK}$?
- If $LK = 46$, what is NM ?
- If $JK = 5x + 20$ and $NO = 20$, what is the value of x ?



Do you UNDERSTAND?

- Vocabulary** How does the term *midsegment* describe the segments discussed in this lesson?
- Reasoning** If two noncollinear segments in the coordinate plane have slope 3, what can you conclude?
- Error Analysis** A student sees this figure and concludes that $\overline{PL} \parallel \overline{NO}$. What is the error in the student's reasoning?

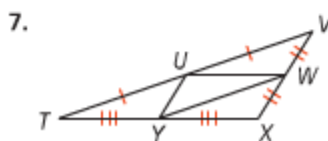


Practice and Problem-Solving Exercises

Practice

Identify three pairs of parallel segments in each diagram.

See Problem 1.



Name the segment that is parallel to the given segment.

9. $\overline{AB}$

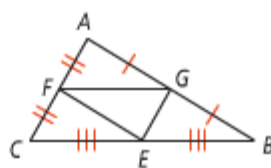
10. $\overline{BC}$

11. $\overline{EF}$

12. $\overline{CA}$

13. $\overline{GE}$

14. $\overline{FG}$



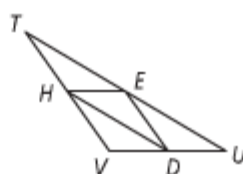
Points E, D, and H are the midpoints of the sides of $\triangle TUV$. $UV = 80$, $TV = 100$, and $HD = 80$.

15. Find HE.

16. Find ED.

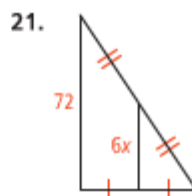
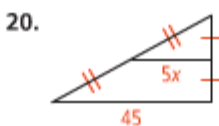
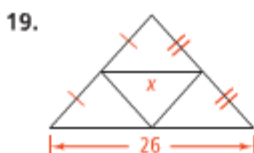
17. Find TU.

18. Find TE.

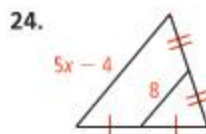
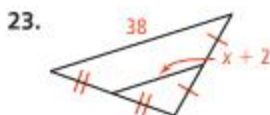
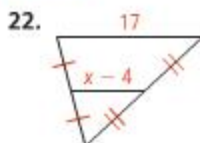


See Problem 2.

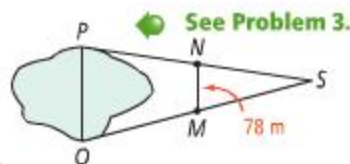
Algebra Find the value of x .



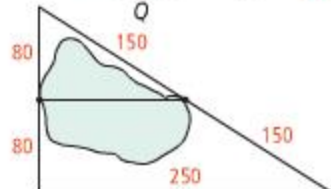
Algebra Find the value of x .



25. **Surveying** A surveyor needs to measure the distance PQ across the lake. Beginning at point S , she locates the midpoints of $\overline{SQ}$ and $\overline{SP}$ at M and N . She then measures $\overline{NM}$. What is PQ ?



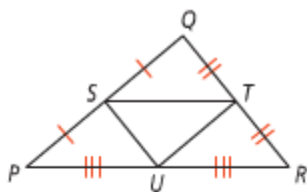
26. **Kayaking** You want to paddle your kayak across a lake. To determine how far you must paddle, you pace out a triangle, counting the number of strides, as shown.
- If your strides average 3.5 ft, what is the length of the longest side of the triangle?
 - What distance must you paddle across the lake?



27. **Architecture** The triangular face of the Rock and Roll Hall of Fame in Cleveland, Ohio, is isosceles. The length of the base is 229 ft 6 in. Each leg is divided into four congruent parts by the red segments. What is the length of the white segment? Explain your reasoning.

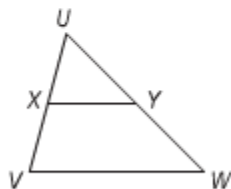


28. **Think About a Plan** Draw $\triangle ABC$. Construct another triangle so that the three sides of $\triangle ABC$ are the midsegments of the new triangle.
- Can you visualize or sketch the final figure?
 - Which segments in your final construction will be parallel?
29. **Writing** In the figure at the right, $m\angle QST = 40$. What is $m\angle QPR$? Explain how you know.
30. **Coordinate Geometry** The coordinates of the vertices of a triangle are $E(1, 2)$, $F(5, 6)$, and $G(3, -2)$.
- Find the coordinates of H , the midpoint of $\overline{EG}$, and J , the midpoint of $\overline{FG}$.
 - Show that $\overline{HJ} \parallel \overline{EF}$.
 - Show that $HJ = \frac{1}{2}EF$.



X is the midpoint of $\overline{UV}$. Y is the midpoint of $\overline{UW}$.

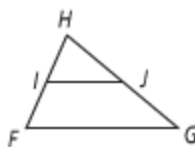
- If $m\angle UXY = 60$, find $m\angle V$.
- If $m\angle W = 45$, find $m\angle UYX$.
- If $XY = 50$, find VW .
- If $VW = 110$, find XY .



$\overline{IJ}$ is a midsegment of $\triangle FGH$. $IJ = 7$, $FH = 10$, and $GH = 13$. Find the perimeter of each triangle.

35. $\triangle IJH$

36. $\triangle FGH$



37. **Kite Design** You design a kite to look like the one at the right. Its diagonals measure 64 cm and 90 cm. You plan to use ribbon, represented by the purple rectangle, to connect the midpoints of its sides. How much ribbon do you need?

(A) 77 cm

(C) 154 cm

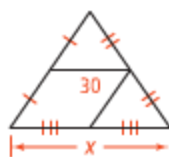
(B) 122 cm

(D) 308 cm

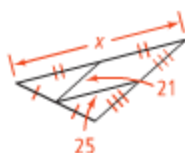


Algebra Find the value of each variable.

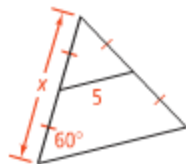
38.



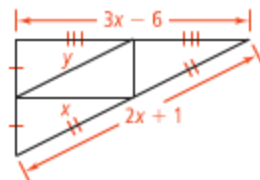
39.



40.



41.

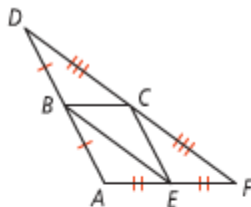


Use the figure at the right for Exercises 42–44.

42. $DF = 24$, $BC = 6$, and $DB = 8$. Find the perimeter of $\triangle ADF$.

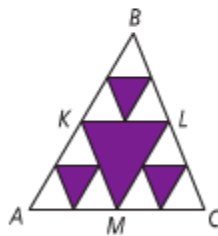
43. **Algebra** If $BE = 2x + 6$ and $DF = 5x + 9$, find DF .

44. **Algebra** If $EC = 3x - 1$ and $AD = 5x + 7$, find EC .



45. **Open-Ended** Explain how you could use the Triangle Midsegment Theorem as the basis for this construction: Draw $\overline{CD}$. Draw point A not on $\overline{CD}$. Construct $\overline{AB}$ so that $\overline{AB} \parallel \overline{CD}$ and $AB = \frac{1}{2}CD$.

46. **Reasoning** In the diagram at the right, K , L , and M are the midpoints of the sides of $\triangle ABC$. The vertices of the three small purple triangles are the midpoints of the sides of $\triangle KBL$, $\triangle AKM$, and $\triangle MLC$. The perimeter of $\triangle ABC$ is 24 cm. What is the perimeter of the shaded region?



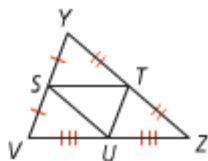
Challenge

47. **Coordinate Geometry** In $\triangle GHJ$, $K(2, 3)$ is the midpoint of $\overline{GH}$, $L(4, 1)$ is the midpoint of $\overline{HJ}$, and $M(6, 2)$ is the midpoint of $\overline{GJ}$. Find the coordinates of G , H , and J .

48. Complete the Prove statement and then write a proof.

Given: In $\triangle VYZ$, S , T , and U are midpoints.

Prove: $\triangle YST \cong \triangle TUZ \cong \triangle SVU \cong ?$



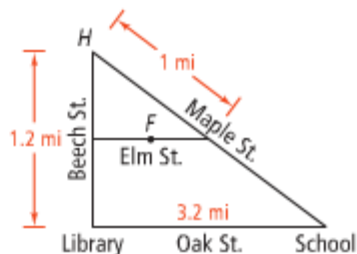
Standardized Test Prep

GRIDDED RESPONSE

SAT/ACT

Use the figure at the right for Exercises 49 and 50. Your home is at point H . Your friend lives at point F , the midpoint of Elm Street. Elm Street intersects Beech Street and Maple Street at their midpoints.

49. Your friend walks to school by going east on Elm and then turning right on Maple. How far in miles does she walk?
50. You walk your dog along this route: Walk from home to Elm along Maple. Walk west on Elm to Beech, south on Beech to the library, and east on Oak to school. Then walk back home along Maple. How far in miles do you walk?



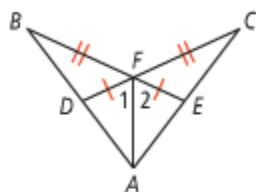
For Exercises 51 and 52, $\triangle ABC$ is a triangle in which $m\angle A = 30$ and $m\angle B = 70$. P , Q , and R are the midpoints of $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$, respectively.

51. What is the measure, in degrees, of $\angle RPQ$?
52. If $QP = 2x + 17$ and $CA = x + 97$, what is CA ?

Mixed Review

Use the figure at the right for Exercises 53 and 54.

53. List all the pairs of congruent triangles that you can find in the figure.
54. **Given:** $\overline{FD} \cong \overline{FE}$, $\overline{BF} \cong \overline{CF}$, $\angle 1 \cong \angle 2$
Prove: $\overline{AB} \cong \overline{AC}$



See Lesson 4-7.

Get Ready! To prepare for Lesson 5-2, do Exercises 55-57.

$\overline{TM}$ bisects $\angle STU$ so that $m\angle STM = 5x + 4$ and $m\angle MTU = 6x - 2$.

See Lesson 1-5.

55. Find the value of x .
56. Find $m\angle STU$.
57. Draw acute $\angle E$. Construct the bisector of $\angle E$.

See Lesson 1-6.

5-2

Perpendicular and Angle Bisectors

Objective To use properties of perpendicular bisectors and angle bisectors



Confused? Try drawing a diagram to "straighten" yourself out.



Getting Ready!

You hang a bulletin board over your desk using string. The bulletin board is crooked. When you straighten the bulletin board, what type of triangle does the string form with the top of the board? How do you know? Visualize the vertical line along the wall that passes through the nail. What relationships exist between this line and the top edge of the straightened bulletin board? Explain.



Dynamic Activity
Perpendicular and Angle Bisectors

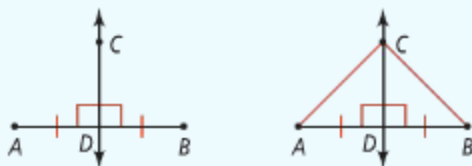
Lesson Vocabulary

- equidistant
- distance from a point to a line

In the Solve It, you thought about the relationships that must exist in order for a bulletin board to hang straight. You will explore these relationships in this lesson.

Essential Understanding There is a special relationship between the points on the perpendicular bisector of a segment and the endpoints of the segment.

In the diagram below on the left, $\overleftrightarrow{CD}$ is the perpendicular bisector of $\overline{AB}$. $\overleftrightarrow{CD}$ is perpendicular to $\overline{AB}$ at its midpoint. In the diagram on the right, $\overline{CA}$ and $\overline{CB}$ are drawn to complete $\triangle CAD$ and $\triangle CBD$.



You should recognize from your work in Chapter 4 that $\triangle CAD \cong \triangle CBD$. So you can conclude that $\overline{CA} \cong \overline{CB}$, or that $CA = CB$. A point is **equidistant** from two objects if it is the same distance from the objects. So point C is equidistant from points A and B.

This suggests a proof of Theorem 5-2, the Perpendicular Bisector Theorem. Its converse is also true and is stated as Theorem 5-3.

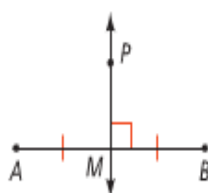
Theorem 5-2 Perpendicular Bisector Theorem

Theorem

If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

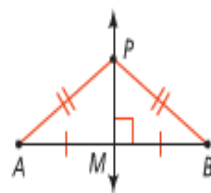
If ...

$\overleftrightarrow{PM} \perp \overline{AB}$ and $MA = MB$



Then ...

$PA = PB$



You will prove Theorem 5-2 in Exercise 32.

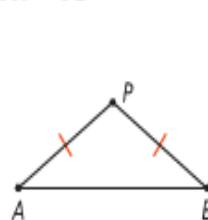
Theorem 5-3 Converse of the Perpendicular Bisector Theorem

Theorem

If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.

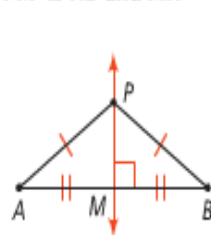
If ...

$PA = PB$



Then ...

$\overleftrightarrow{PM} \perp \overline{AB}$ and $MA = MB$



You will prove Theorem 5-3 in Exercise 33.



Problem 1 Using the Perpendicular Bisector Theorem

Algebra What is the length of $\overline{AB}$?

$\overline{BD}$ is the perpendicular bisector of $\overline{AC}$, so B is equidistant from A and C.

$$BA = BC \quad \text{Perpendicular Bisector Theorem}$$

$$4x = 6x - 10 \quad \text{Substitute } 4x \text{ for } BA \text{ and } 6x - 10 \text{ for } BC.$$

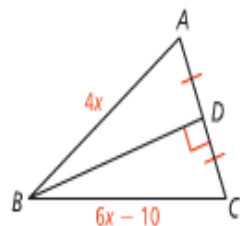
$$-2x = -10 \quad \text{Subtract } 6x \text{ from each side.}$$

$$x = 5 \quad \text{Divide each side by } -2.$$

Now find AB.

$$AB = 4x$$

$$AB = 4(5) = 20 \quad \text{Substitute 5 for } x.$$



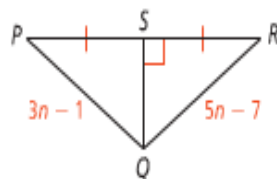
Think

How do you know $\overline{BD}$ is the perpendicular bisector of $\overline{AC}$?

The markings in the diagram show that $\overline{BD}$ is perpendicular to $\overline{AC}$ at the midpoint of $\overline{AC}$.



Got It? 1. What is the length of $\overline{QR}$?



Plan

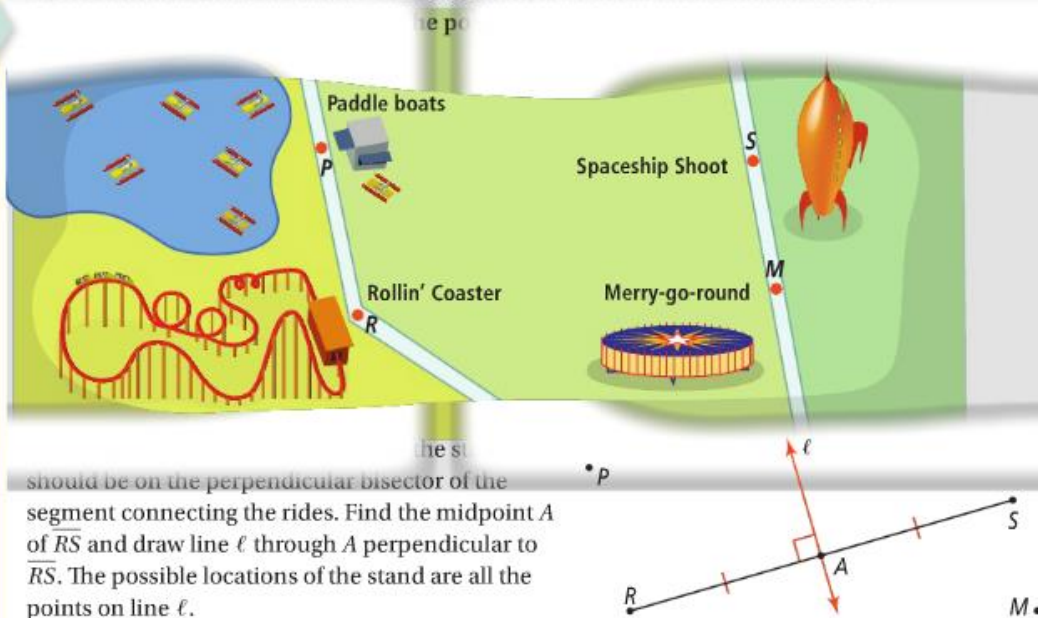
How do you find points that are equidistant from two given points?

By the Converse of the Perpendicular Bisector Theorem, points equidistant from two given points are on the perpendicular bisector of the segment that joins the two points.



Problem 2 Using a Perpendicular Bisector

A park director wants to build a T-shirt stand equidistant from the Rollin' Coaster



should be on the perpendicular bisector of the segment connecting the rides. Find the midpoint A of $\overline{RS}$ and draw line ℓ through A perpendicular to $\overline{RS}$. The possible locations of the stand are all the points on line ℓ .

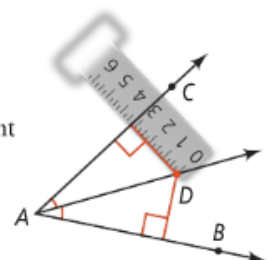
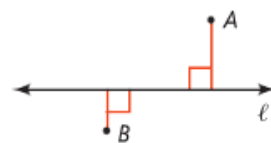


- Got It?** 2. a. Suppose the director wants the T-shirt stand to be equidistant from the paddle boats and the Spaceship Shoot. What are the possible locations?
b. **Reasoning** Can you place the T-shirt stand so that it is equidistant from the paddle boats, the Spaceship Shoot, and the Rollin' Coaster? Explain.

Essential Understanding There is a special relationship between the points on the bisector of an angle and the sides of the angle.

The **distance from a point to a line** is the length of the perpendicular segment from the point to the line. This distance is also the length of the shortest segment from the point to the line. You will prove this in Lesson 5-6. In the figure at the right, the distances from A to ℓ and from B to ℓ are represented by the red segments.

In the diagram, $\overrightarrow{AD}$ is the bisector of $\angle CAB$. If you measure the lengths of the perpendicular segments from D to the two sides of the angle, you will find that the lengths are equal. Point D is equidistant from the sides of the angle.



take note

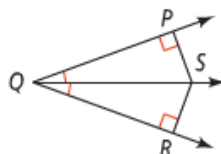
Theorem 5-4 Angle Bisector Theorem

Theorem

If a point is on the bisector of an angle, then the point is equidistant from the sides of the angle.

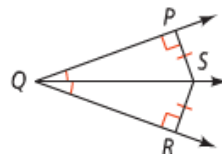
If ...

$\overrightarrow{QS}$ bisects $\angle PQR$, $\overline{SP} \perp \overline{QP}$,
and $\overline{SR} \perp \overline{QR}$



Then ...

$SP = SR$



You will prove Theorem 5-4 in Exercise 34.

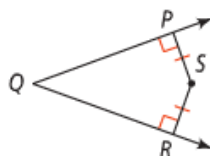
Theorem 5-5 Converse of the Angle Bisector Theorem

Theorem

If a point in the interior of an angle is equidistant from the sides of the angle, then the point is on the angle bisector.

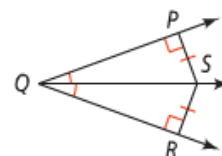
If ...

$\overline{SP} \perp \overline{QP}$, $\overline{SR} \perp \overline{QR}$,
and $SP = SR$



Then ...

$\overrightarrow{QS}$ bisects $\angle PQR$



You will prove Theorem 5-5 in Exercise 35.



Problem 3 Using the Angle Bisector Theorem

Algebra What is the length of $\overline{RM}$?

Know

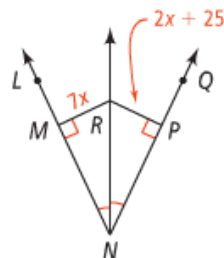
$\overrightarrow{NR}$ bisects $\angle LNQ$.
 $\overline{RM} \perp \overline{NL}$ and $\overline{RP} \perp \overline{NQ}$.

Need

The length of $\overline{RM}$

Plan

Use the Angle Bisector Theorem to write an equation you can solve for x .



$$\begin{aligned} RM &= RP && \text{Angle Bisector Theorem} \\ 7x &= 2x + 25 && \text{Substitute.} \\ 5x &= 25 && \text{Subtract } 2x \text{ from each side.} \\ x &= 5 && \text{Divide each side by } 5. \end{aligned}$$

Now find RM .

$$\begin{aligned} RM &= 7x \\ &= 7(5) = 35 && \text{Substitute } 5 \text{ for } x. \end{aligned}$$

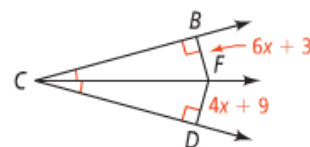
Think

How can you use the expression given for RP to check your answer?

Substitute 5 for x in the expression $2x + 25$ and verify that the result is 35.



Got It? 3. What is the length of $\overline{FB}$?



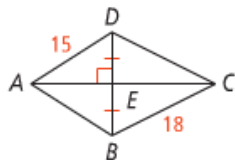


Lesson Check

Do you know HOW?

Use the figure at the right for Exercises 1–3.

1. What is the relationship between $\overline{AC}$ and $\overline{BD}$?
2. What is the length of $\overline{AB}$?
3. What is the length of $\overline{DC}$?



Do you UNDERSTAND?

4. **Vocabulary** Draw a line and a point not on the line. Draw the segment that represents the distance from the point to the line.
5. **Writing** Point P is in the interior of $\angle LOX$. Describe how you can determine whether P is on the bisector of $\angle LOX$ without drawing the angle bisector.



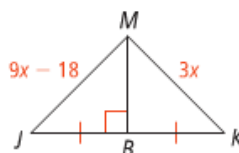
Practice and Problem-Solving Exercises



Practice

Use the figure at the right for Exercises 6–8.

6. What is the relationship between $\overline{MB}$ and $\overline{JK}$?
7. What is value of x ?
8. Find JM .



See Problem 1.

Reading Maps For Exercises 9 and 10, use the map of a part of Manhattan.

See Problem 2.

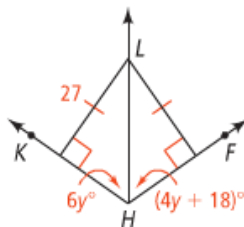
9. Which school is equidistant from the subway stations at Union Square and 14th Street? How do you know?
10. Is St. Vincent's Hospital equidistant from Village Kids Nursery School and Legacy School? How do you know?
11. **Writing** On a piece of paper, mark a point H for home and a point S for school. Describe how to find the set of points equidistant from H and S .



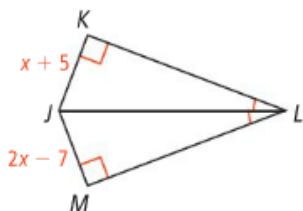
Use the figure at the right for Exercises 12–15.

See Problem 3.

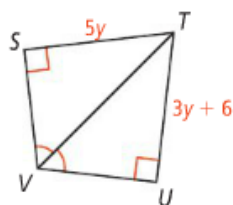
12. According to the diagram, how far is L from $\overline{HK}$? From $\overline{HF}$?
13. How is $\overline{HL}$ related to $\angle KHF$? Explain.
14. Find the value of y .
15. Find $m\angle KHL$ and $m\angle FHL$.



16. **Algebra** Find x , JK , and JM .

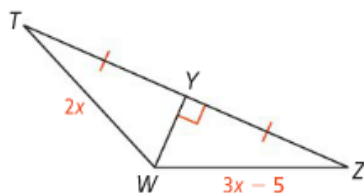


17. **Algebra** Find y , ST , and TU .

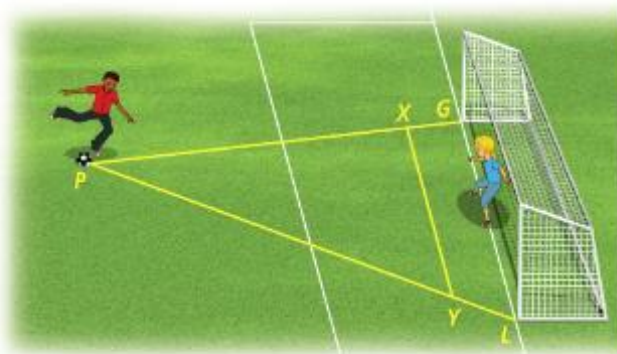


Algebra Use the figure at the right for Exercises 18–22.

18. Find the value of x .
 19. Find TW .
 20. Find WZ .
 21. What kind of triangle is $\triangle TWZ$? Explain.
 22. If R is on the perpendicular bisector of $\overline{TZ}$, then R is $\underline{\hspace{1cm}}$ from T and Z , or $\underline{\hspace{1cm}} = \underline{\hspace{1cm}}$.

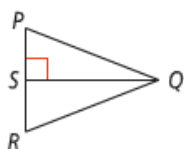


23. **Think About a Plan** In the diagram at the right, the soccer goalie will prepare for a shot from the player at point P by moving out to a point on $\overline{XY}$. To have the best chance of stopping the ball, should the goalie stand at the point on $\overline{XY}$ that lies on the perpendicular bisector of $\overline{GL}$ or at the point on $\overline{XY}$ that lies on the bisector of $\angle GPL$? Explain your reasoning.

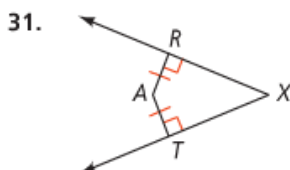
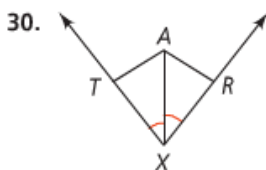
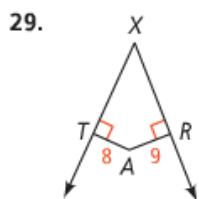


- How can you draw a diagram to help?
 - Would the goalie want to be the same distance from G and L or from $\overline{PG}$ and $\overline{PL}$?
24. a. **Constructions** Draw a large triangle, $\triangle CDE$. Construct the angle bisectors of each angle.
 b. **Make a Conjecture** What appears to be true about the angle bisectors?
 c. Test your conjecture with another triangle.
25. a. **Constructions** Draw a large acute scalene triangle, $\triangle PQR$. Construct the perpendicular bisectors of each side.
 b. **Make a Conjecture** What appears to be true about the perpendicular bisectors?
 c. Test your conjecture with another triangle.
26. Write Theorems 5-2 and 5-3 as a single biconditional statement.
27. Write Theorems 5-4 and 5-5 as a single biconditional statement.

28. **Error Analysis** To prove that $\triangle PQR$ is isosceles, a student began by stating that since Q is on the segment perpendicular to $\overline{PR}$, Q is equidistant from the endpoints of $\overline{PR}$. What is the error in the student's reasoning?



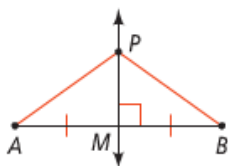
Writing Determine whether A must be on the bisector of $\angle TXR$. Explain.



32. **Proof** Prove the Perpendicular Bisector Theorem.

Given: $\overleftrightarrow{PM} \perp \overline{AB}$, $\overleftrightarrow{PM}$ bisects $\overline{AB}$

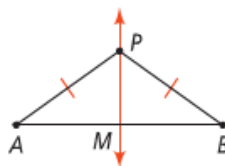
Prove: $AP = BP$



33. **Proof** Prove the Converse of the Perpendicular Bisector Theorem.

Given: $PA = PB$ with $\overleftrightarrow{PM} \perp \overline{AB}$ at M .

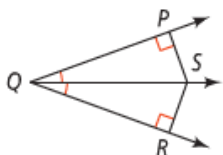
Prove: P is on the perpendicular bisector of $\overline{AB}$.



34. **Proof** Prove the Angle Bisector Theorem.

Given: $\overleftrightarrow{QS}$ bisects $\angle PQR$,
 $\overline{SP} \perp \overline{QP}$, $\overline{SR} \perp \overline{QR}$

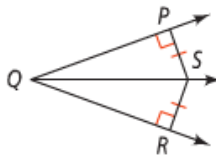
Prove: $SP = SR$



35. **Proof** Prove the Converse of the Angle Bisector Theorem.

Given: $\overline{SP} \perp \overline{QP}$, $\overline{SR} \perp \overline{QR}$,
 $SP = SR$

Prove: $\overleftrightarrow{QS}$ bisects $\angle PQR$.

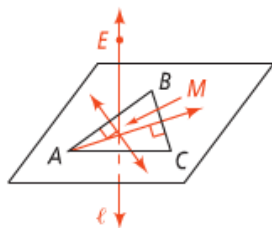


36. **Coordinate Geometry** Use points $A(6, 8)$, $O(0, 0)$, and $B(10, 0)$.

- Write equations of lines ℓ and m such that $\ell \perp \overrightarrow{OA}$ at A and $m \perp \overrightarrow{OB}$ at B .
- Find the intersection C of lines ℓ and m .
- Show that $CA = CB$.
- Explain why C is on the bisector of $\angle AOB$.

Challenge

37. A , B , and C are three noncollinear points. Describe and sketch a line in plane ABC such that points A , B , and C are equidistant from the line. Justify your response.
38. **Reasoning** M is the intersection of the perpendicular bisectors of two sides of $\triangle ABC$. Line ℓ is perpendicular to plane ABC at M . Explain why a point E on ℓ is equidistant from A , B , and C . (Hint: See page 48, Exercise 33. Explain why $\triangle EAM \cong \triangle EBM \cong \triangle ECM$.)

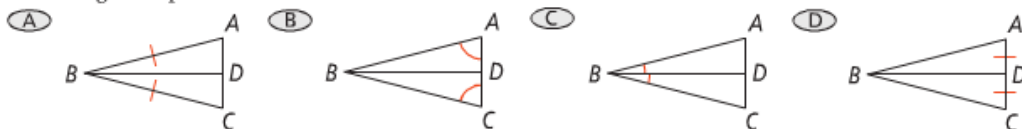


Standardized Test Prep

SAT/ACT

39. For $A(1, 3)$ and $B(1, 9)$, which point lies on the perpendicular bisector of $\overline{AB}$?
 (A) $(3, 3)$ (B) $(1, 5)$ (C) $(6, 6)$ (D) $(3, 12)$
40. What is the converse of the following conditional statement?
 If a triangle is isosceles, then it has two congruent angles.
 (F) If a triangle is isosceles, then it has two congruent sides.
 (G) If a triangle has congruent sides, then it is equilateral.
 (H) If a triangle has two congruent angles, then it is isosceles.
 (I) If a triangle is not isosceles, then it does not have two congruent angles.

41. Which figure represents the statement $\overline{BD}$ bisects $\angle ABC$?

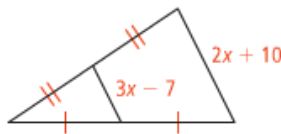


Short Response

42. The line $y = 7$ is the perpendicular bisector of the segment with endpoints $A(2, 10)$ and $B(2, k)$. What is the value of k ? Explain your reasoning.

Mixed Review

43. Find the value of x in the figure at the right.
44. $\angle 1$ and $\angle 2$ are complementary and $\angle 1$ and $\angle 3$ are supplementary. If $m\angle 2 = 30$, what is $m\angle 3$?



See Lesson 5-1.

See Lesson 1-5.

Get Ready! To prepare for Lesson 5-3, do Exercises 45–47.

45. What is the slope of a line that is perpendicular to the line $y = -3x + 4$?
46. Line ℓ is a horizontal line. What is the slope of a line perpendicular to ℓ ?
47. Describe the line $x = 5$.

See Lesson 3-8.

Concept Byte

Use With Lesson 5-3

ACTIVITY

Paper Folding Bisectors

In Activity 1, you will use paper folding to investigate the bisectors of the angles of a triangle.

Activity 1

- Step 1** Draw and cut out three different triangles: one acute, one right, and one obtuse.
- Step 2** Use paper folding to make the angle bisectors of each angle of your acute triangle. What do you notice about the angle bisectors?
- Step 3** Repeat Step 2 with your right triangle and your obtuse triangle. Does your discovery from Step 2 still hold true?

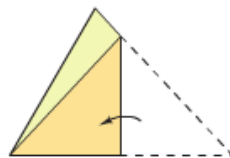


Folding an angle bisector

In Activity 2, you will use paper folding to investigate the perpendicular bisectors of the sides of a triangle.

Activity 2

- Step 1** Draw and cut out two different triangles: one acute and one right.
- Step 2** Use paper folding to make the perpendicular bisector of each side of your acute triangle. What do you notice about the perpendicular bisectors?
- Step 3** Repeat Step 1 with your right triangle. Does your discovery from Step 2 still



Folding a perpendicular

Exercises

- 1. **Make a Conjecture** Make a conjecture about the bisectors of the angles of a triangle.
- 2. **Make a Conjecture** Make a conjecture about the perpendicular bisectors of the sides of a triangle.
- 3. **Extend** Draw and cut out an obtuse triangle. Fold the perpendicular bisectors.
 - a. How do the results for your obtuse triangle compare to the results for your acute and right triangles from Activity 2?
 - b. Based on your answer to part (a), how would you revise your conjecture in Exercise 2?
- 4. **Extend** For what type of triangle would the three perpendicular bisectors and the three angle bisectors intersect at the same point?

5-3

Bisectors in Triangles



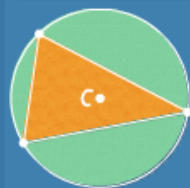
The lines you drew to each side of the triangle aren't just regular perpendicular lines!

Objective To identify properties of perpendicular bisectors and angle bisectors



Getting Ready!

Construct a circle and label its center C . Choose any three points on the circle and connect them to form a triangle. Draw three lines from C such that each line is perpendicular to one side of the triangle. What conjecture can you make about the two segments into which each side of the triangle is divided? Justify your reasoning.



Dynamic Activity

Bisectors in Triangles



Lesson Vocabulary

- concurrent
- point of concurrency
- circumcenter of a triangle
- circumscribed about
- incenter of a triangle
- inscribed in

In the Solve It, the three lines you drew intersect at one point, the center of the circle. When three or more lines intersect at one point, they are **concurrent**. The point at which they intersect is the **point of concurrency**.

Essential Understanding For any triangle, certain sets of lines are always concurrent. Two of these sets of lines are the perpendicular bisectors of the triangle's three sides and the bisectors of the triangle's three angles.

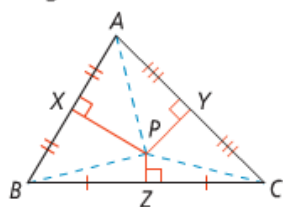
take note

Theorem 5-6 Concurrency of Perpendicular Bisectors Theorem

Theorem

The perpendicular bisectors of the sides of a triangle are concurrent at a point equidistant from the vertices.

Diagram



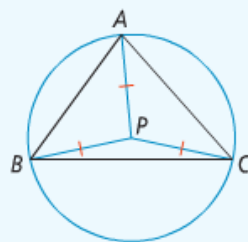
Symbols

Perpendicular bisectors $\overline{PX}$, $\overline{PY}$, and $\overline{PZ}$ are concurrent at P .

$$PA = PB = PC$$

The point of concurrency of the perpendicular bisectors of a triangle is called the **circumcenter of the triangle**.

Since the circumcenter is equidistant from the vertices, you can use the circumcenter as the center of the circle that contains each vertex of the triangle. You say the circle is **circumscribed about** the triangle.

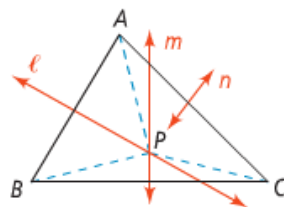


Proof **Proof of Theorem 5-6**

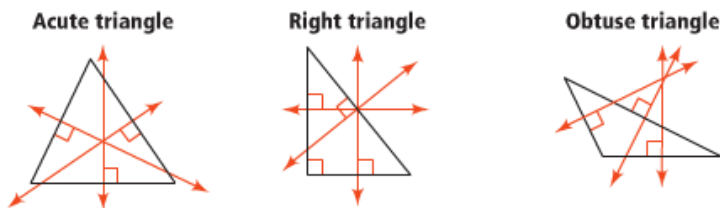
Given: Lines ℓ , m , and n are the perpendicular bisectors of the sides of $\triangle ABC$. P is the intersection of lines ℓ and m .

Prove: Line n contains point P , and $PA = PB = PC$.

Proof: A point on the perpendicular bisector of a segment is equidistant from the endpoints of the segment. Point P is on ℓ , which is the perpendicular bisector of $\overline{AB}$, so $PA = PB$. Using the same reasoning, since P is on m , and m is the perpendicular bisector of $\overline{BC}$, $PB = PC$. Thus, $PA = PC$ by the Transitive Property. Since $PA = PC$, P is equidistant from the endpoints of $\overline{AC}$. Then, by the converse of the Perpendicular Bisector Theorem, P is on line n , the perpendicular bisector of $\overline{AC}$.



The circumcenter of a triangle can be inside, on, or outside a triangle.

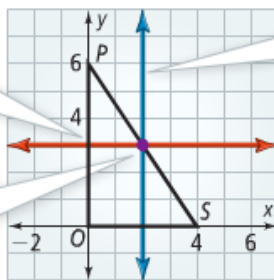
**Problem 1** **Finding the Circumcenter of a Triangle**

What are the coordinates of the circumcenter of the triangle with vertices $P(0, 6)$, $O(0, 0)$, and $S(4, 0)$?

Find the intersection point of two of the triangle's perpendicular bisectors. Here, it is easiest to find the perpendicular bisectors of $\overline{PO}$ and $\overline{OS}$.

Step 1 $(0, 3)$ is the midpoint of $\overline{PO}$. The line through $(0, 3)$ that is perpendicular to $\overline{PO}$ is $y = 3$.

Step 3 Find the point where the two perpendicular bisectors intersect. $x = 2$ and $y = 3$ intersect at $(2, 3)$.



Step 2 $(2, 0)$ is the midpoint of $\overline{OS}$. The line through $(2, 0)$ that is perpendicular to $\overline{OS}$ is $x = 2$.

The coordinates of the circumcenter of the triangle are $(2, 3)$.



Got It? 1. What are the coordinates of the circumcenter of the triangle with vertices $A(2, 7)$, $B(10, 7)$, and $C(10, 3)$?

Think

Does the location of the circumcenter make sense?

Yes, $\triangle POS$ is a right triangle, so its circumcenter should lie on its hypotenuse.

Think

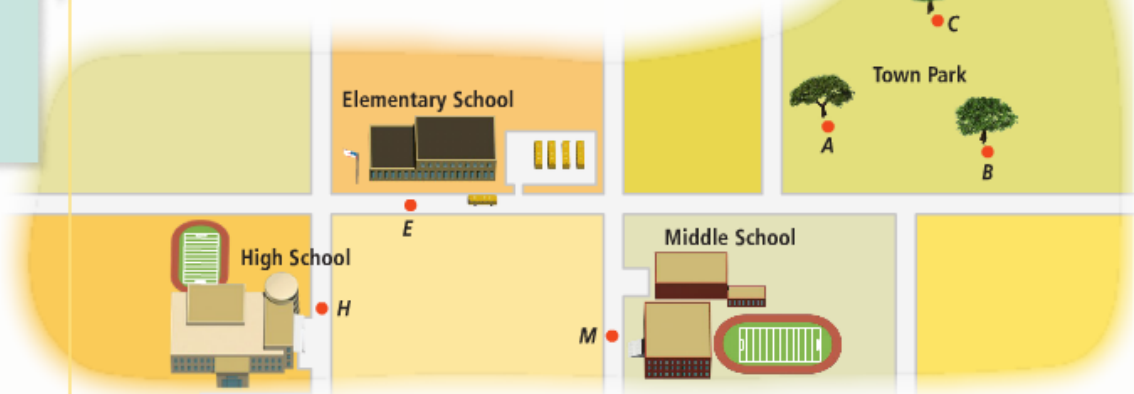
How do you find a point equidistant from three points?

As long as the three points are noncollinear, they are vertices of a triangle. Find the circumcenter of the triangle.

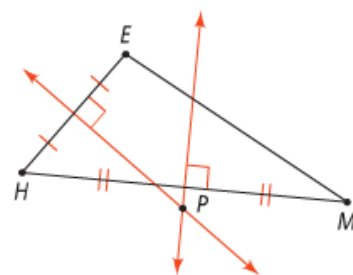


Problem 2 Using a Circumcenter

A town planner wants to locate a new fire station equidistant from the elementary, middle, and high schools. Where should he locate the station?



The three schools form the vertices of a triangle. The planner should locate the fire station at P , the point of concurrency of the perpendicular bisectors of $\triangle EMH$. This point is the circumcenter of $\triangle EMH$ and is equidistant from the three schools at E , M , and H .



Got It? 2. In Problem 2, the town planner wants to place a bench equidistant from the three trees in the park. Where should he place the bench?

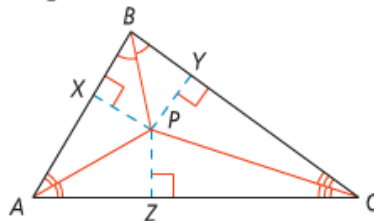
Take note

Theorem 5-7 Concurrency of Angle Bisectors Theorem

Theorem

The bisectors of the angles of a triangle are concurrent at a point equidistant from the sides of the triangle.

Diagram



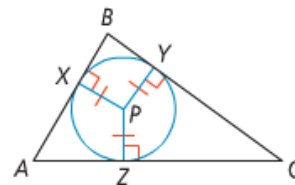
Symbols

Angle bisectors $\overline{AP}$, $\overline{BP}$, and $\overline{CP}$ are concurrent at P .

$$PX = PY = PZ$$

You will prove Theorem 5-7 in Exercise 24.

The point of concurrency of the angle bisectors of a triangle is called the **incenter of the triangle**. For any triangle, the incenter is always inside the triangle. In the diagram, points X , Y , and Z are equidistant from P , the incenter of $\triangle ABC$. P is the center of the circle that is **inscribed** in the triangle.





Problem 3 Identifying and Using the Incenter of a Triangle

Algebra $GE = 2x - 7$ and $GF = x + 4$. What is GD ?

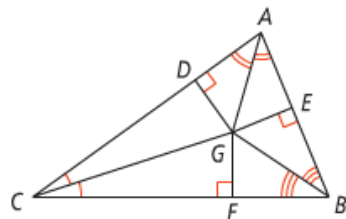
G is the incenter of $\triangle ABC$ because it is the point of concurrency of the angle bisectors. By the Concurrency of Angle Bisectors Theorem, the distances from the incenter to the three sides of the triangle are equal, so $GE = GF = GD$. Use this relationship to find x .

$$\begin{array}{rcl} 2x - 7 & = & x + 4 \quad GE = GF \\ 2x & = & x + 11 \quad \text{Add 7 to each side.} \\ x & = & 11 \quad \text{Subtract } x \text{ from each side.} \end{array}$$

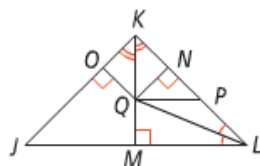
Now find GF .

$$\begin{array}{rcl} GF & = & x + 4 \\ & = & 11 + 4 = 15 \quad \text{Substitute 11 for } x. \end{array}$$

Since $GF = GD$, $GD = 15$.



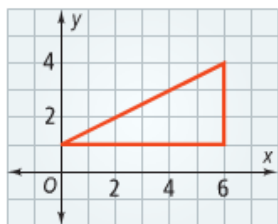
- Got It?** 3. a. $QN = 5x + 36$ and $QM = 2x + 51$. What is QO ?
b. **Reasoning** Is it possible for QP to equal 50? Explain.



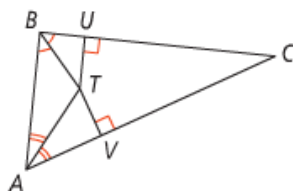
Lesson Check

Do you know HOW?

1. What are the coordinates of the circumcenter of the following triangle?

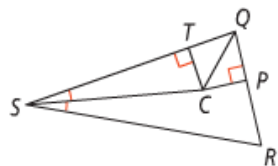


2. In the figure at the right, $TV = 3x - 12$ and $TU = 5x - 24$. What is the value of x ?



Do you UNDERSTAND?

3. **Vocabulary** A triangle's circumcenter is outside the triangle. What type of triangle is it?
4. **Reasoning** You want to find the circumcenter of a triangle. Why do you only need to find the intersection of two of the triangle's perpendicular bisectors, instead of all three?
5. **Error Analysis** Your friend sees the triangle at the right and concludes that $CT = CP$. What is the error in your friend's reasoning?
6. **Compare and Contrast** How are the circumcenter and incenter of a triangle alike? How are they different?

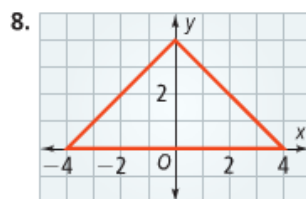
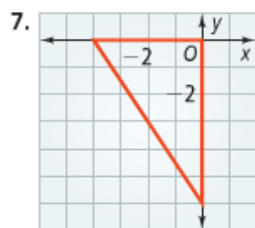


Practice and Problem-Solving Exercises

A Practice

Coordinate Geometry Find the coordinates of the circumcenter of each triangle.

See Problem 1.



Coordinate Geometry Find the coordinates of the circumcenter of $\triangle ABC$.

9. $A(0, 0)$
 $B(3, 0)$
 $C(3, 2)$

10. $A(0, 0)$
 $B(4, 0)$
 $C(4, -3)$

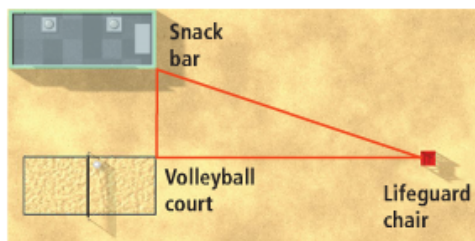
11. $A(-4, 5)$
 $B(-2, 5)$
 $C(-2, -2)$

12. $A(-1, -2)$
 $B(-5, -2)$
 $C(-1, -7)$

13. $A(1, 4)$
 $B(1, 2)$
 $C(6, 2)$

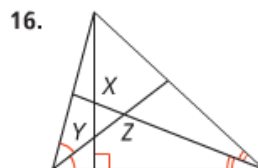
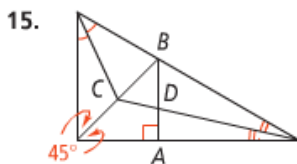
14. **City Planning** Copy the diagram of the beach. Show where town officials should place a recycling barrel so that it is equidistant from the lifeguard chair, the snack bar, and the volleyball court. Explain.

See Problem 2.

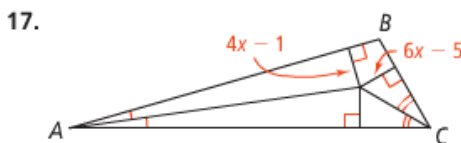


Name the point of concurrency of the angle bisectors.

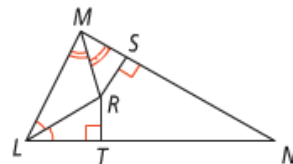
See Problem 3.



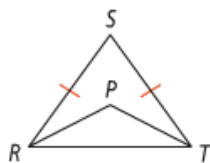
Find the value of x .



18. $RS = 4(x - 3) + 6$ and $RT = 5(2x - 6)$.



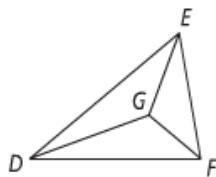
19. **Think About a Plan** In the figure at the right, P is the incenter of isosceles $\triangle RST$. What type of triangle is $\triangle RPT$? Explain.
- What segments determine the incenter of a triangle?
 - What do you know about the base angles of an isosceles triangle?



Constructions Draw a triangle that fits the given description. Then construct the inscribed circle and the circumscribed circle. Describe your method.

20. right triangle, $\triangle DEF$
 21. obtuse triangle, $\triangle STU$

22. **Algebra** In the diagram at the right, G is the incenter of $\triangle DEF$, $m\angle DEF = 60^\circ$, and $m\angle EFD = 2 \cdot m\angle EDF$. What are $m\angle DGE$, $m\angle DGF$, and $m\angle EGF$?



23. **Writing** Ivars found an old piece of paper inside an antique book. It read,

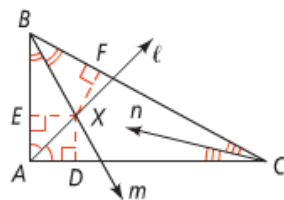
From the spot I buried Olaf's treasure, equal sets of paces did I measure; each of three directions in a line, there to plant a seedling Norway pine. I could not return for failing health; now the hounds of Haiti guard my wealth. —Karl

After searching Caribbean islands for five years, Ivars found an island with three tall Norway pines. How might Ivars find where Karl buried Olaf's treasure?

24. Use the diagram at the right to prove the Concurrency of Angle Bisectors Theorem.

Given: Rays ℓ , m , and n are bisectors of the angles of $\triangle ABC$. X is the intersection of rays ℓ and m , $\overline{XD} \perp \overline{AC}$, $\overline{XE} \perp \overline{AB}$, and $\overline{XF} \perp \overline{BC}$.

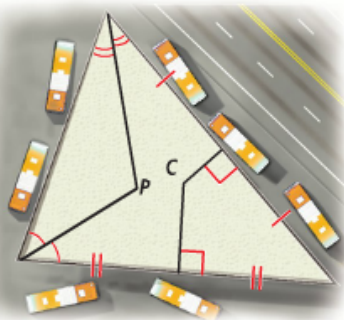
Prove: Ray n contains point X , and $XD = XE = XF$.



25. **Noise Control** You are trying to talk to a friend on the phone in a busy bus station. The buses are so loud that you can hardly hear. Referring to the figure at the right, should you stand at P or C to be as far as possible from all the buses? Explain.

Reasoning Determine whether each statement is *true* or *false*. If the statement is false, give a counterexample.

26. The incenter of a triangle is equidistant from all three vertices.
 27. The incenter of a triangle always lies inside the triangle.
 28. You can circumscribe a circle about any three points in a plane.
 29. If point C is the circumcenter of $\triangle PQR$ and the circumcenter of $\triangle PQS$, then R and S must be the same point.



- Challenge** 30. **Reasoning** Explain why the circumcenter of a right triangle is on one of the triangle's sides.

Determine whether each statement is *always*, *sometimes*, or *never* true. Explain.

31. It is possible to find a point equidistant from three parallel lines in a plane.
32. The circles inscribed in and circumscribed about an isosceles triangle have the same center.

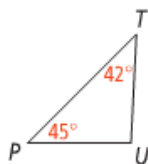
Standardized Test Prep

SAT/ACT

33. Which of the following statements is *false*?
- (A) The bisectors of the angles of a triangle are concurrent.
 - (B) The midsegments of a triangle are concurrent.
 - (C) The perpendicular bisectors of the sides of a triangle are concurrent.
 - (D) Four lines intersecting in one point are concurrent.

34. What type of triangle is $\triangle PUT$?

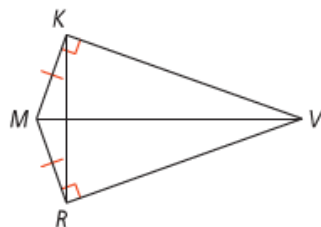
- (F) right isosceles
- (G) acute isosceles
- (H) obtuse scalene
- (I) acute scalene



35. Which statement is logically equivalent to the following statement?

If a triangle is right isosceles, then it has exactly two acute angles.

- (A) If a triangle is right isosceles, then it has one right angle.
- (B) If a triangle has exactly two acute angles, then it is right isosceles.
- (C) If a triangle does not have exactly two acute angles, then it is not right isosceles.
- (D) If a triangle is not right isosceles, then it does not have a right angle.



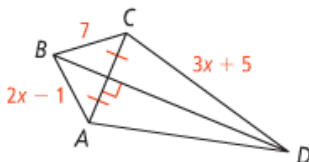
Short Response

36. Refer to the figure at the right above. Explain in two different ways why $\overline{MV}$ is the angle bisector of $\angle KVR$.

Mixed Review

Use the figure at the right for Exercises 37 and 38.

37. Find the value of x .
38. Find the length of $\overline{AD}$.



See Lesson 5-2.

Get Ready! To prepare for Lesson 5-4, do Exercises 39 and 40.

Find the coordinates of the midpoint of $\overline{AB}$ with the given endpoints.

39. $A(3, 0)$, $B(3, 16)$

40. $A(6, 8)$, $B(4, -1)$

See Lesson 1-7.

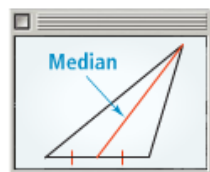
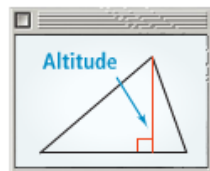
Special Segments in Triangles

You already know about two sets of lines that are concurrent for any triangle. In the following activity, you will use geometry software to confirm what you know about the concurrency of a triangle's perpendicular bisectors and angle bisectors. Then you will explore two more sets of special segments in triangles.

Activity

Use geometry software.

- Construct a triangle and the three perpendicular bisectors of its sides. Use your result to confirm Theorem 5-6, the Concurrency of Perpendicular Bisectors Theorem.
- Construct a triangle and its three angle bisectors. Use your result to confirm Theorem 5-7, the Concurrency of Angle Bisectors Theorem.
- An *altitude* of a triangle is the perpendicular segment from a vertex to the line containing the opposite side. Construct a triangle. Through a vertex of the triangle construct a segment that is perpendicular to the line containing the side opposite that vertex. Next construct the altitudes from the other two vertices.
- A *median* of a triangle is the segment joining the midpoint of a side and the opposite vertex. Construct a triangle. Construct the midpoint of one side. Draw the median. Then construct the other two medians.



Exercises

- What property do the lines containing altitudes and the medians seem to have? Does the property still hold as you manipulate the triangles?
- State your conjectures about the lines containing altitudes and about the medians of a triangle.
- Copy the table. Think about acute, right, and obtuse triangles. Use *inside*, *on*, or *outside* to describe the location of each point of concurrency.

	Perpendicular Bisectors	Angle Bisectors	Lines Containing the Altitudes	Medians
Acute Triangle	■	■	■	■
Right Triangle	■	■	■	■
Obtuse Triangle	■	■	■	■

- Extend** What observations, if any, can you make about these special segments for isosceles triangles? For equilateral triangles?

5-4

Medians and Altitudes

Objective To identify properties of medians and altitudes of a triangle

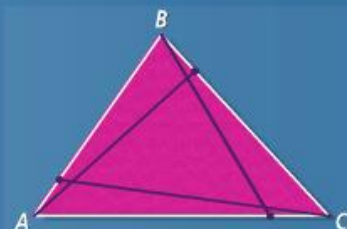


You can use different colors for the sets of segments so you can see the pattern more easily.



Getting Ready!

Draw a large acute scalene $\triangle ABC$. On each side, mark the point that is $\frac{1}{5}$ of the distance from one of the side's endpoints, as shown in the diagram. Connect each of these points to the opposite vertex. Repeat this process for $\frac{1}{4}$ and $\frac{1}{3}$. What do you think the result will be for $\frac{1}{2}$? Check your answer. Were you correct?



Dynamic Activity

Medians and Altitudes

In the Solve It, the last set of segments you drew are the triangle's medians. A **median of a triangle** is a segment whose endpoints are a vertex and the midpoint of the opposite side.



Essential Understanding A triangle's three medians are always concurrent.

Lesson Vocabulary

- median of a triangle
- centroid of a triangle
- altitude of a triangle
- orthocenter of a triangle

Take note

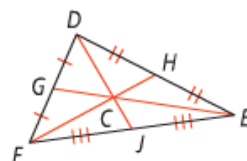
Theorem 5-8 Concurrency of Medians Theorem

The medians of a triangle are concurrent at a point that is two thirds the distance from each vertex to the midpoint of the opposite side.

$$DC = \frac{2}{3}DJ$$

$$EC = \frac{2}{3}EG$$

$$FC = \frac{2}{3}FH$$



You will prove Theorem 5-8 in Lesson 6-9.

In a triangle, the point of concurrency of the medians is the **centroid of the triangle**. The point is also called the *center of gravity* of a triangle because it is the point where a triangular shape will balance. For any triangle, the centroid is always inside the triangle.

Plan

How do you use the centroid?

Write an equation relating the length of the whole median to the length of the segment from the vertex to the centroid.



Problem 1 Finding the Length of a Median

GRIDDED RESPONSE

In the diagram at the right, $XA = 8$. What is the length of $\overline{XB}$?

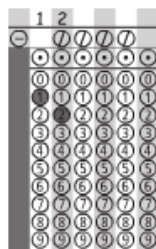
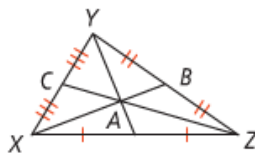
A is the centroid of $\triangle XYZ$ because it is the point of concurrency of the triangle's medians.

$$XA = \frac{2}{3}XB \quad \text{Concurrency of Medians Theorem}$$

$$8 = \frac{2}{3}XB \quad \text{Substitute 8 for } XA.$$

$$\left(\frac{3}{2}\right)8 = \left(\frac{3}{2}\right)\frac{2}{3}XB \quad \text{Multiply each side by } \frac{3}{2}.$$

$$12 = XB \quad \text{Simplify.}$$



- Got It?** 1. a. In the diagram for Problem 1, $ZA = 9$. What is the length of $\overline{ZC}$?
b. **Reasoning** What is the ratio of ZA to AC ? Explain.

An **altitude of a triangle** is the perpendicular segment from a vertex of the triangle to the line containing the opposite side. An altitude of a triangle can be inside or outside the triangle, or it can be a side of the triangle.

Plan

How do you determine whether a segment is an altitude or a median? Look at whether the segment is perpendicular to a side (altitude) and/or bisects a side (median).



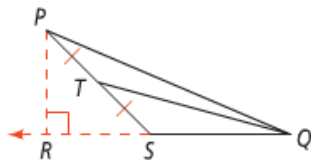
Problem 2 Identifying Medians and Altitudes

- A** For $\triangle PQS$, is $\overline{PR}$ a median, an altitude, or neither? Explain.

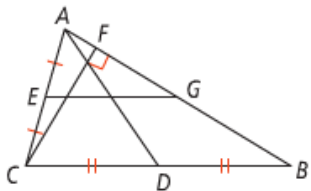
$\overline{PR}$ is a segment that extends from vertex P to the line containing $\overline{SQ}$, the side opposite P . $\overline{PR} \perp \overline{QR}$, so $\overline{PR}$ is an altitude of $\triangle PQS$.

- B** For $\triangle PQS$, is $\overline{QT}$ a median, an altitude, or neither? Explain.

$\overline{QT}$ is a segment that extends from vertex Q to the side opposite Q . Since $\overline{PT} \cong \overline{TS}$, T is the midpoint of $\overline{PS}$. So $\overline{QT}$ is a median of $\triangle PQS$.



- Got It?** 2. For $\triangle ABC$, is each segment a median, an altitude, or neither? Explain.
a. $\overline{AD}$ b. $\overline{EG}$ c. $\overline{CF}$



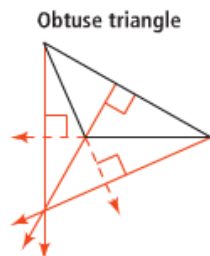
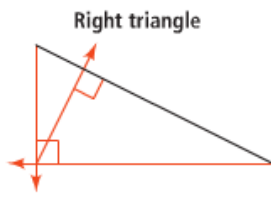
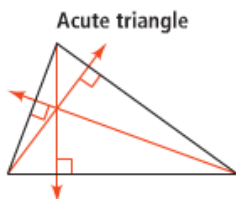
take note

Theorem 5-9 Concurrency of Altitudes Theorem

The lines that contain the altitudes of a triangle are concurrent.

You will prove Theorem 5-9 in Lesson 6-9.

The lines that contain the altitudes of a triangle are concurrent at the **orthocenter of the triangle**. The orthocenter of a triangle can be inside, on, or outside the triangle.



Problem 3 Finding the Orthocenter

$\triangle ABC$ has vertices $A(1, 3)$, $B(2, 7)$, and $C(6, 3)$. What are the coordinates of the orthocenter of $\triangle ABC$?

Know

The coordinates of the three vertices

Need

The intersection point of the triangle's altitudes

Plan

Write the equations of the lines that contain two of the altitudes. Then solve the system of equations.

Think

Which two altitudes should you choose?

It does not matter, but the altitude to $\overline{AC}$ is a vertical line, so its equation will be easy to find.

Step 1 Find the equation of the line containing the altitude to $\overline{AC}$. Since $\overline{AC}$ is horizontal, the line containing the altitude to $\overline{AC}$ is vertical. The line passes through the vertex $B(2, 7)$. The equation of the line is $x = 2$.

Step 2 Find the equation of the line containing the altitude to $\overline{BC}$. The slope of the line containing $\overline{BC}$ is $\frac{3-7}{6-2} = -1$. Since the product of the slopes of two perpendicular lines is -1 , the line containing the altitude to $\overline{BC}$ has slope 1.

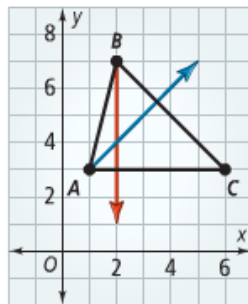
The line passes through the vertex $A(1, 3)$. The equation of the line is $y - 3 = 1(x - 1)$, which simplifies to $y = x + 2$.

Step 3 Find the orthocenter by solving this system of equations: $x = 2$
 $y = x + 2$

$$y = 2 + 2 \quad \text{Substitute 2 for } x \text{ in the second equation.}$$

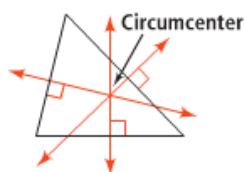
$$y = 4 \quad \text{Simplify.}$$

The coordinates of the orthocenter are $(2, 4)$.

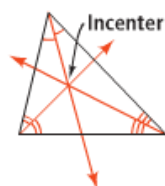


Got It? 3. $\triangle DEF$ has vertices $D(1, 2)$, $E(1, 6)$, and $F(4, 2)$. What are the coordinates of the orthocenter of $\triangle DEF$?

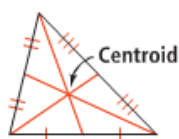
Perpendicular Bisectors



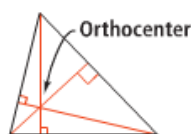
Angle Bisectors



Medians



Altitudes

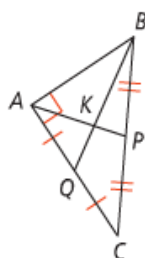


Lesson Check

Do you know HOW?

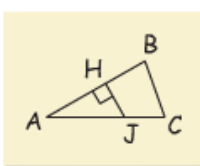
Use $\triangle ABC$ for Exercises 1–4.

1. Is $\overline{AP}$ a median or an altitude?
2. If $AP = 18$, what is KP ?
3. If $BK = 15$, what is KQ ?
4. Which two segments are altitudes?



Do you UNDERSTAND?

5. **Error Analysis** Your classmate says she drew $\overline{HJ}$ as an altitude of $\triangle ABC$. What error did she make?
6. **Reasoning** Does it matter which two altitudes you use to locate the orthocenter of a triangle? Explain.
7. **Reasoning** The orthocenter of $\triangle ABC$ lies at vertex A. What can you conclude about $\overline{BA}$ and $\overline{AC}$? Explain.

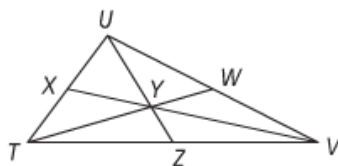


Practice and Problem-Solving Exercises

A Practice

In $\triangle TUV$, Y is the centroid.

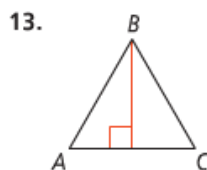
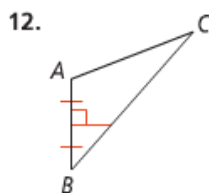
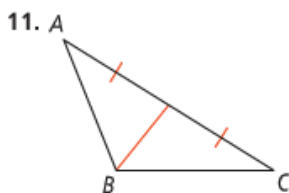
8. If $YW = 9$, find TY and TW .
9. If $YU = 9$, find ZY and ZU .
10. If $VX = 9$, find VY and YX .



See Problem 1.

For $\triangle ABC$, is the red segment a median, an altitude, or neither? Explain.

See Problem 2.



14. $A(0, 0)$
 $B(4, 0)$
 $C(4, 2)$

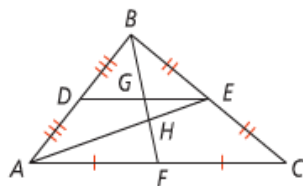
15. $A(2, 6)$
 $B(8, 6)$
 $C(6, 2)$

16. $A(0, -2)$
 $B(4, -2)$
 $C(-2, -8)$

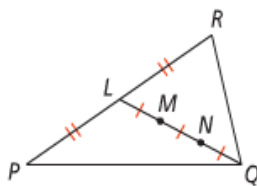


Name the centroid.

17.

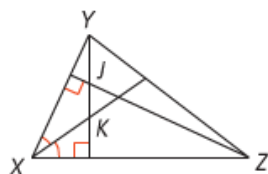


18.

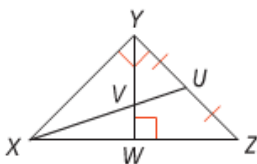


Name the orthocenter of $\triangle XYZ$.

19.

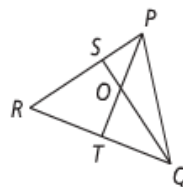


20.



21. **Think About a Plan** In the diagram at the right, $\overline{QS}$ and $\overline{PT}$ are altitudes and $m\angle R = 55$. What is $m\angle POQ$?

- What does it mean for a segment to be an altitude?
- What do you know about the sum of the angle measures in a triangle?
- How do you sketch overlapping triangles separately?



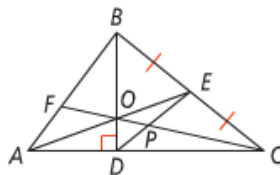
Constructions Draw a triangle that fits the given description. Then construct the centroid and the orthocenter.

22. acute scalene triangle, $\triangle LMN$

23. obtuse isosceles triangle, $\triangle RST$

In Exercises 24–27, name each segment.

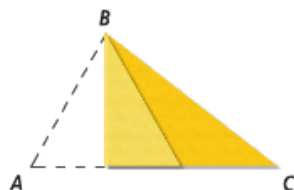
24. a median in $\triangle ABC$
 25. an altitude in $\triangle ABC$
 26. a median in $\triangle BDC$
 27. an altitude in $\triangle AOC$



28. **Reasoning** A centroid separates a median into two segments. What is the ratio of the length of the shorter segment to the length of the longer segment?

Paper Folding The figures below show how to construct altitudes and medians by paper folding. Refer to them for Exercises 29 and 30.

Folding an Altitude

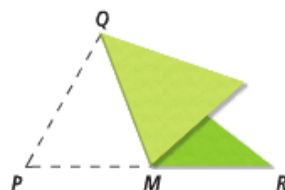


Fold the triangle so that a side $\overline{AC}$ overlaps itself and the fold contains the opposite vertex B .

Folding a Median



Fold one vertex R to another vertex P . This locates the midpoint M of a side.



Unfold the triangle. Then fold it so that the fold contains the midpoint M and the opposite vertex Q .

29. Cut out a large triangle. Fold the paper carefully to construct the three medians of the triangle and demonstrate the Concurrency of Medians Theorem. Use a ruler to measure the length of each median and the distance of each vertex from the centroid.

30. Cut out a large acute triangle. Fold the paper carefully to construct the three altitudes of the triangle and demonstrate the Concurrency of Altitudes Theorem.

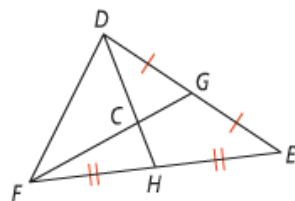
31. In the figure at the right, C is the centroid of $\triangle DEF$.
If $GF = 12x^2 + 6y$, which expression represents CF ?

(A) $6x^2 + 3y$

(C) $8x^2 + 4y$

(B) $4x^2 + 2y$

(D) $8x^2 + 3y$



32. **Reasoning** What type of triangle has its orthocenter on the exterior of the triangle? Draw a sketch to support your answer.

33. **Writing** Explain why the median to the base of an isosceles triangle is also an altitude.

34. **Coordinate Geometry** $\triangle ABC$ has vertices $A(0, 0)$, $B(2, 6)$, and $C(8, 0)$. Complete the following steps to verify the Concurrency of Medians Theorem for $\triangle ABC$.

a. Find the coordinates of midpoints L , M , and N .

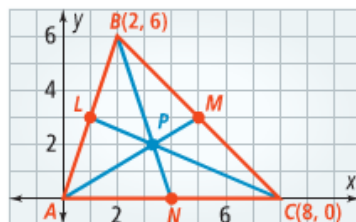
b. Find equations of $\overleftrightarrow{AM}$, $\overleftrightarrow{BN}$, and $\overleftrightarrow{CL}$.

c. Find the coordinates of P , the intersection of $\overleftrightarrow{AM}$ and $\overleftrightarrow{BN}$.

This point is the centroid.

d. Show that point P is on $\overleftrightarrow{CL}$.

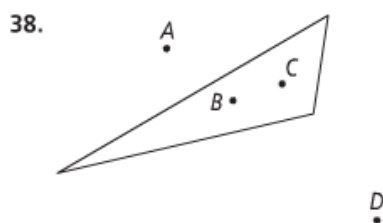
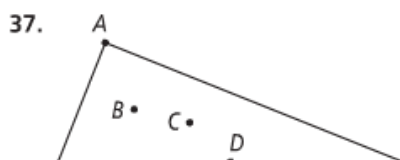
e. Use the Distance Formula to show that point P is two-thirds of the distance from each vertex to the midpoint of the opposite side.



35. **Constructions** A , B , and O are three noncollinear points. Construct point C such that O is the orthocenter of $\triangle ABC$. Describe your method.

36. **Reasoning** In an isosceles triangle, show that the circumcenter, incenter, centroid, and orthocenter can be four different points, but all four must be collinear.

A , B , C , and D are points of concurrency for the triangle. Determine whether each point is a *circumcenter*, *incenter*, *centroid*, or *orthocenter*. Explain.



39. **History** In 1765, Leonhard Euler proved that, for any triangle, three of the four points of concurrency are collinear. The line that contains these three points is known as Euler's Line. Use Exercises 37 and 38 to determine which point of concurrency does not necessarily lie on Euler's Line.

Standardized Test Prep

SAT/ACT

For Exercises 40 and 41, use the figure at the right.

40. If $CR = 24$, what is KR ?

(A) 6

(B) 8

41. If $TR = 12$ what is CP ?

(F) 16

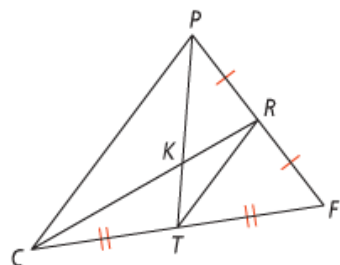
(G) 18

(C) 12

(D) 16

(H) 24

(I) 36

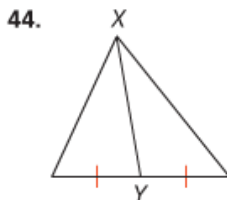
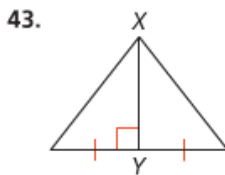


Extended Response

42. The orthocenter of a triangle lies outside the triangle. Where are its circumcenter, incenter, and centroid located in relation to the triangle? Draw and label diagrams to support your answers.

Mixed Review

Is $\overline{XY}$ a *perpendicular bisector*, an *angle bisector*, or *neither*? Explain.



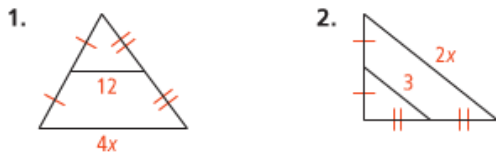
See Lesson 5-3.

Get Ready! To prepare for Lesson 5-5, do Exercises 45–47.

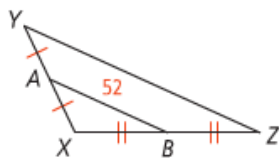
Write the negation of each statement.

See Lesson 2-2.

45. Two angles are congruent. 46. You are not 16 years old. 47. $m\angle A < 90$

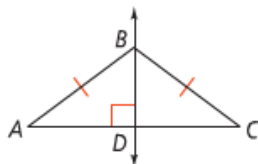
**Do you know HOW?****Algebra** Find the value of x .

Use the figure below for Exercises 3–5.



- Find YZ .
- $AX = 26$ and $BZ = 36$. Find the perimeter of $\triangle XYZ$.
- Which angle is congruent to $\angle XBA$? How do you know?

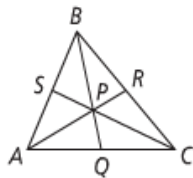
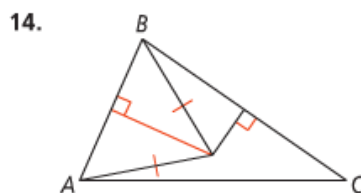
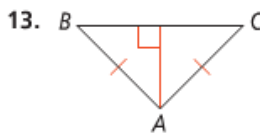
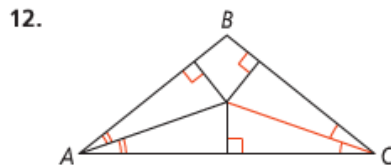
For the figure below, what can you conclude about each of the following? Explain.



- $\angle CDB$
- $\triangle ABD$ and $\triangle CBD$
- $\overline{AD}$ and $\overline{DC}$

In the figure at the right, P is the centroid of $\triangle ABC$.

- If $PR = 6$, find AP and AR .
- If $PB = 6$, find QP and QB .
- If $SC = 6$, find CP and PS .

For $\triangle ABC$, is the red line a *perpendicular bisector*, an *angle bisector*, a *median*, an *altitude*, or *none of these*? Explain.

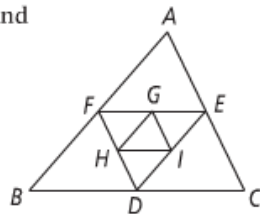
15. $\triangle PQR$ has vertices $P(2, 5)$, $Q(8, 5)$, and $R(8, 1)$. Find the coordinates of the circumcenter and the orthocenter of $\triangle PQR$.

Do you UNDERSTAND?

- Writing** Explain how to construct a median of a triangle and an altitude of a triangle.
- Error Analysis** Point O is the incenter of scalene $\triangle XYZ$. Your friend says that $m\angle YXO = m\angle YZO$. Is your friend correct? Explain.

The sides of $\triangle DEF$ are the midsegments of $\triangle ABC$. The sides of $\triangle GHI$ are the midsegments of $\triangle DEF$.

- Which sides, if any, of $\triangle GHI$ and $\triangle ABC$ are parallel? Explain.
- What are the relationships between the side lengths of $\triangle GHI$ and $\triangle ABC$? Explain.



5-5

Indirect Proof

Objective To use indirect reasoning to write proofs



It's okay to write a number in a square because you know it can't be any other number.



Getting Ready!

The goal of this game is to fill in the empty squares with numbers. The numbers 1, 2, 3, and 4 must appear once in each row and once in each column. Copy and complete the games on a piece of paper.

Game A

1			2
4		1	
	4		
		2	

Game B

			3
	1		
2		1	
	4		

Lesson Vocabulary

- indirect reasoning
- indirect proof

In the Solve It, you can conclude that a square must contain a certain number if you can eliminate the other three numbers as possibilities. This type of reasoning is called indirect reasoning. In **indirect reasoning**, all possibilities are considered and then all but one are proved false. The remaining possibility must be true.

Essential Understanding You can use indirect reasoning as another method of proof.

A proof involving indirect reasoning is an **indirect proof**. Often in an indirect proof, a statement and its negation are the only possibilities. When you see that one of these possibilities leads to a conclusion that contradicts a fact you know to be true, you can eliminate that possibility. For this reason, indirect proof is sometimes called *proof by contradiction*.

Take note

Key Concept Writing an Indirect Proof

- Step 1** State as a temporary assumption the opposite (negation) of what you want to prove.
- Step 2** Show that this temporary assumption leads to a contradiction.
- Step 3** Conclude that the temporary assumption must be false and that what you want to prove must be true.

In the first step of an indirect proof you assume as true the opposite of what you want to prove.



Problem 1 Writing the First Step of an Indirect Proof

Suppose you want to write an indirect proof of each statement. As the first step of the proof, what would you assume?

- A** An integer n is divisible by 5.

The opposite of “is divisible by” is “is not divisible by.”

Assume temporarily that n is not divisible by 5.

- B** You do not have soccer practice today.

The opposite of “do not have” is “do have.”

Assume temporarily that you do have soccer practice today.



- Got It?** 1. Suppose you want to write an indirect proof of each statement. As the first step of the proof, what would you assume?

a. $\triangle BOX$ is not acute.

b. At least one pair of shoes you bought cost more than \$25.

To write an indirect proof, you have to be able to identify a contradiction.



Problem 2 Identifying Contradictions

Which two statements contradict each other?

I. $\overline{FG} \parallel \overline{KL}$

II. $\overline{FG} \cong \overline{KL}$

III. $\overline{FG} \perp \overline{KL}$

Segments can be parallel and congruent. Statements I and II do not contradict each other.

Segments can be congruent and perpendicular. Statements II and III do not contradict each other.

Parallel segments do not intersect, so they cannot be perpendicular.

Statements I and III contradict each other.



- Got It?** 2. a. Which two statements contradict each other?

I. $\triangle XYZ$ is acute.

II. $\triangle XYZ$ is scalene.

III. $\triangle XYZ$ is equiangular.

- b. **Reasoning** Statements I and II below contradict each other.

Statement III is the negation of Statement I. Are

Statements II and III equivalent? Explain your reasoning.

I. $\triangle ABC$ is scalene.

II. $\triangle ABC$ is equilateral.

III. $\triangle ABC$ is not scalene.

Think

How do you find the opposite of a statement?

Write the negation of the statement. This often involves adding or removing the word *not*.

Think

How do you know that two statements contradict each other?

A statement contradicts another statement if it is impossible for both to be true at the same time.

Problem 3 Writing an Indirect Proof

Given: $\triangle ABC$ is scalene.

Prove: $\angle A$, $\angle B$, and $\angle C$ all have different measures.

Think

Assume temporarily the opposite of what you want to prove.

Show that this assumption leads to a contradiction.

Conclude that the temporary assumption must be false and that what you want to prove must be true.

Write

Assume temporarily that two angles of $\triangle ABC$ have the same measure. Assume that $m\angle A = m\angle B$.

By the Converse of the Isosceles Triangle Theorem, the sides opposite $\angle A$ and $\angle B$ are congruent. This contradicts the given information that $\triangle ABC$ is scalene.

The assumption that two angles of $\triangle ABC$ have the same measure must be false. Therefore, $\angle A$, $\angle B$, and $\angle C$ all have different measures.



Got It? 3. **Given:** $7(x + y) = 70$ and $x \neq 4$.

Prove: $y \neq 6$



Lesson Check

Do you know HOW?

- Suppose you want to write an indirect proof of the following statement. As the first step of the proof, what would you assume?
Quadrilateral $ABCD$ has four right angles.
- Write a statement that contradicts the following statement. Draw a diagram to support your answer.
Lines a and b are parallel.

Do you UNDERSTAND?

- Error Analysis** A classmate began an indirect proof as shown below. Explain and correct your classmate's error.

~~Given: $\triangle ABC$~~

~~Prove: $\angle A$ is obtuse.~~

~~Assume temporarily that $\angle A$ is acute.~~



Practice and Problem-Solving Exercises



Practice

Write the first step of an indirect proof of the given statement.

- | | | |
|----------------------------------|--|----------------------------------|
| 4. It is raining outside. | 5. $\angle J$ is not a right angle. | 6. $\triangle PEN$ is isosceles. |
| 7. At least one angle is obtuse. | 8. $\overline{XY} \cong \overline{AB}$ | 9. $m\angle 2 > 90$ |

 See Problem 1.

Identify the two statements that contradict each other.

See Problem 2.

10. I. $\triangle PQR$ is equilateral.
 II. $\triangle PQR$ is a right triangle.
 III. $\triangle PQR$ is isosceles.
11. I. $\ell \parallel m$
 II. ℓ and m do not intersect.
 III. ℓ and m are skew.
12. I. Each of the two items that Val bought costs more than \$10.
 II. Val spent \$34 for the two items.
 III. Neither of the two items that Val bought costs more than \$15.
13. I. In right $\triangle ABC$, $m\angle A = 60$.
 II. In right $\triangle ABC$, $\angle A \cong \angle C$.
 III. In right $\triangle ABC$, $m\angle B = 90$.

14. **Developing Proof** Fill in the blanks to prove the following statement.

See Problem 3.

If the Yoga Club and Go Green Club together have fewer than 20 members and the Go Green Club has 10 members, then the Yoga Club has fewer than 10 members.

Given: The total membership of the Yoga Club and the Go Green Club is fewer than 20. The Go Green Club has 10 members.

Prove: The Yoga Club has fewer than 10 members.

Proof: Assume temporarily that the Yoga Club has 10 or more members.

This means that together the two clubs have a. ? members.

This contradicts the given information that b. ?.

The temporary assumption is false. Therefore, it is true that c. ?.

15. **Developing Proof** Fill in the blanks to prove the following statement.

In a given triangle, $\triangle LMN$, there is at most one right angle.

Given: $\triangle LMN$

Prove: $\triangle LMN$ has at most one right angle.

Proof: Assume temporarily that $\triangle LMN$ has more than one a. ?. That is, assume that both $\angle M$ and $\angle N$ are b. ?. If $\angle M$ and $\angle N$ are both right angles, then $m\angle M = m\angle N =$ c. ?. By the Triangle Angle-Sum Theorem, $m\angle L + m\angle M + m\angle N =$ d. ?. Use substitution to write the equation $m\angle L +$ e. ? + f. ? = 180. When you solve for $m\angle L$, you find that $m\angle L =$ g. ?. This means that there is no $\triangle LMN$, which contradicts the given statement. So the temporary assumption that $\triangle LMN$ has h. ? must be false. Therefore, $\triangle LMN$ has i. ?.



16. **History** Use indirect reasoning to eliminate all but one of the following answers.

In what year was George Washington born?

(A) 1492

(B) 1732

(C) 1902

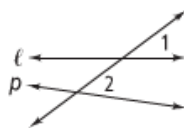
(D) 2002

17. **Think About a Plan** Write an indirect proof.

Given: $\angle 1 \neq \angle 2$

Prove: $\ell \not\parallel p$

- What assumption should be the first step of your proof?
- In the figure, what type of angle pair do $\angle 1$ and $\angle 2$ form?



Write the first step of an indirect proof of the given statement.

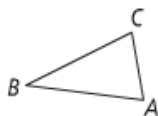
18. If a number n ends in 5, then it is not divisible by 2.
19. If point X is on the perpendicular bisector of $\overline{AB}$, then $\overline{XB} \cong \overline{XA}$.
20. If a transversal intersects two parallel lines, then alternate exterior angles are congruent.
21. **Reasoning** Identify the two statements that contradict each other.
 - I. The orthocenter of $\triangle JRK$ is on the triangle.
 - II. The centroid of $\triangle JRK$ is inside the triangle.
 - III. $\triangle JRK$ is an obtuse triangle.

Write an indirect proof.

22. Use the figure at the right.

Given: $\triangle ABC$ with $BC > AC$

Prove: $\angle A \cong \angle B$



23. **Given:** $\triangle XYZ$ is isosceles.

Prove: Neither base angle is a right angle.

Writing For Exercises 24 and 25, write a convincing argument that uses indirect reasoning.

24. **Chemistry** Ice is forming on the sidewalk in front of Toni's house. Show that the temperature of the sidewalk surface must be 32°F or lower.
25. Show that an obtuse triangle cannot contain a right angle.
26. **Error Analysis** Your friend wants to prove indirectly that $\triangle ABC$ is equilateral. For a first step, he writes, "Assume temporarily that $\triangle ABC$ is scalene." What is wrong with your friend's statement? How can he correct himself?
27. **Literature** In Arthur Conan Doyle's story "The Sign of the Four," Sherlock Holmes talks to his friend Watson about how a culprit enters a room that has only four entrances: a door, a window, a chimney, and a hole in the roof.

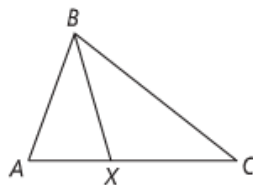
"You will not apply my precept," he said, shaking his head. "How often have I said to you that when you have eliminated the impossible, whatever remains, however improbable, must be the truth? We know that he did not come through the door, the window, or the chimney. We also know that he could not have been concealed in the room, as there is no concealment possible. Whence, then, did he come?"

How did the culprit enter the room? Explain.

28. **Open-Ended** Describe a real-life situation in which you used an indirect argument to convince someone of your point of view. Outline your argument.

**Challenge**

Use the figure at the right for Exercises 29 and 30.

29. Given: $\triangle ABC$ is scalene, $m\angle ABX = 36$, $m\angle CBX = 36$ **Proof:** $\overline{XB}$ is not perpendicular to $\overline{AC}$.**30. Given:** $\triangle ABC$ is scalene, $m\angle ABX = 36$, $m\angle CBX = 36$ **Proof:** $\overline{AX} \neq \overline{XC}$ **Standardized Test Prep****SAT/ACT****31.** What temporary assumption is the first step of the following indirect proof?**Given:** The sides of $\triangle SFK$ measure 3 cm, 4 cm, and 5 cm.**Prove:** The orthocenter of $\triangle SFK$ is on the triangle.

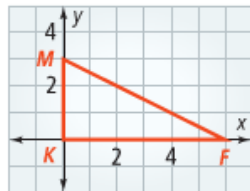
- (A) The incenter of $\triangle SFK$ is inside the triangle.
 (B) The orthocenter of $\triangle SFK$ is inside the triangle.
 (C) The centroid of $\triangle SFK$ is outside the triangle.
 (D) The orthocenter of $\triangle SFK$ is inside or outside the triangle.

32. $\triangle LMN \cong \triangle OPQ$, $m\angle L = 39$, and $m\angle P = 61$. What is $m\angle Q$?

- (F) 39 (G) 80 (H) 100 (I) 141

33. In the diagram, what are the coordinates of the circumcenter of $\triangle KMF$?

- (A) (1.5, 1.5) (C) (3, 1.5)
 (B) (6, 0) (D) (0, 0)

**Short Response****34.** For what types of triangles are the centroid, circumcenter, incenter, and orthocenter all inside the triangle? Explain.**Mixed Review****35.** The distances from the centroid of a triangle to its vertices are 16 cm, 17 cm, and 18 cm. What is the length of the shortest median? **See Lesson 5-4.****36.** The orthocenter of isosceles $\triangle ABC$ lies outside the triangle and $m\angle A = 30$. What are the measures of the two other angles?**37.** You think, "If I leave home at 7:10, I'll catch the 7:25 bus. If I catch the 7:25 bus, I'll get to school before class starts. I am leaving and it's 7:10, so I'll get to school on time." Which law of deductive reasoning are you using? **See Lesson 2-4.****Get Ready!** To prepare for Lesson 5-6, do Exercises 38–40.Graph $\triangle ABC$. List the sides in order from shortest to longest. **See Lesson 1-7.****38.** $A(5, 0)$, $B(0, 8)$, $C(0, 0)$ **39.** $A(2, 4)$, $B(-5, 1)$, $C(0, 0)$ **40.** $A(3, 0)$, $B(4, 3)$, $C(8, 0)$

The solutions of an inequality are all the numbers that make the inequality true. The following chart reviews the Properties of Inequality.

take note

Key Concept Properties of Inequality

Addition Property

For all real numbers a , b , c , and d :

If $a > b$ and $c \geq d$, then $a + c > b + d$.

Multiplication Property

If $a > b$ and $c > 0$, then $ac > bc$.

If $a > b$ and $c < 0$, then $ac < bc$.

Transitive Property

If $a > b$ and $b > c$, then $a > c$.

You can use the Addition and Multiplication Properties of Inequality to solve inequalities.

Example

Algebra Solve $-6x + 7 > 25$.

$$-6x + 7 > 25$$

$$-6x > 18 \quad \text{Subtract 7 from each side.}$$

$$\frac{-6x}{-6} < \frac{18}{-6} \quad \text{Divide each side by } -6. \text{ Remember to reverse the inequality symbol.}$$

$$x < -3 \quad \text{Simplify.}$$

Exercises

Algebra Solve each inequality.

1. $7x - 13 \leq -20$

2. $3x + 8 > 16$

3. $-2x - 5 < 16$

4. $8y + 2 \geq 14$

5. $a + 1 \leq 91$

6. $-x - 2 > 17$

7. $-4z - 10 < -12$

8. $9x - 8 \geq 82$

9. $6n + 3 \leq -18$

10. $c + 13 > 34$

11. $3x - 5x + 2 < 12$

12. $2(y - 5) > -24$

13. $-3(4x - 1) \geq 15$

14. $-n - 27 \leq 92$

15. $8x - 4 + x > -76$

16. $8y - 4y + 11 \leq -33$

17. $x + 78 \geq -284$

18. $4(5a + 3) < -8$

5-6

Inequalities in One Triangle

Objective To use inequalities involving angles and sides of triangles

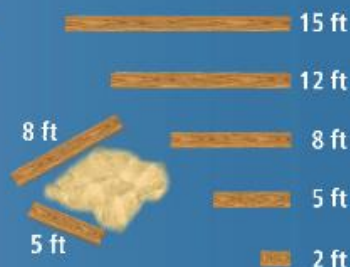


Think about whether the shape of the triangle would be easy to play in.



Getting Ready!

For a neighborhood improvement project, you volunteer to help build a new sandbox at the town playground. You have two boards that will make up two sides of the triangular sandbox. One is 5 ft long and the other is 8 ft long. Boards come in the lengths shown. Which boards can you use for the third side of the sandbox? Explain.



Dynamic Activity
Triangle Inequalities

In the Solve It, you explored triangles formed by various lengths of board. You may have noticed that changing the angle formed by two sides of the sandbox changes the length of the third side.

Essential Understanding The angles and sides of a triangle have special relationships that involve inequalities.

Take note

Property Comparison Property of Inequality

If $a = b + c$ and $c > 0$, then $a > b$.

Proof Proof of the Comparison Property of Inequality

Given: $a = b + c, c > 0$

Prove: $a > b$

Statements	Reasons
1) $c > 0$	1) Given
2) $b + c > b + 0$	2) Addition Property of Inequality
3) $b + c > b$	3) Identity Property of Addition
4) $a = b + c$	4) Given
5) $a > b$	5) Substitution

The Comparison Property of Inequality allows you to prove the following corollary to the Triangle Exterior Angle Theorem (Theorem 3-11).

take note

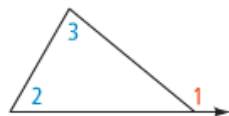
Corollary Corollary to the Triangle Exterior Angle Theorem

Corollary

The measure of an exterior angle of a triangle is greater than the measure of each of its remote interior angles.

If . . .

$\angle 1$ is an exterior angle



Then . . .

$m\angle 1 > m\angle 2$ and
 $m\angle 1 > m\angle 3$

Proof Proof of the Corollary

Given: $\angle 1$ is an exterior angle of the triangle.

Prove: $m\angle 1 > m\angle 2$ and $m\angle 1 > m\angle 3$.

Proof: By the Triangle Exterior Angle Theorem, $m\angle 1 = m\angle 2 + m\angle 3$. Since $m\angle 2 > 0$ and $m\angle 3 > 0$, you can apply the Comparison Property of Inequality and conclude that $m\angle 1 > m\angle 2$ and $m\angle 1 > m\angle 3$.

Think

How do you identify an exterior angle?

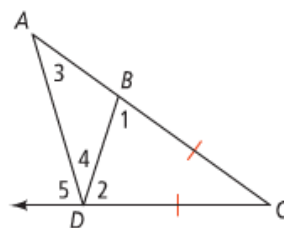
An exterior angle must be formed by the extension of a side of the triangle. Here, $\angle 1$ is an exterior angle of $\triangle ABD$, but $\angle 2$ is not.



Problem 1 Applying the Corollary

Use the figure at the right. Why is $m\angle 2 > m\angle 3$?

In $\triangle ACD$, $\overline{CB} \cong \overline{CD}$, so by the Isosceles Triangle Theorem, $m\angle 1 = m\angle 2$. $\angle 1$ is an exterior angle of $\triangle ABD$, so by the Corollary to the Triangle Exterior Angle Theorem, $m\angle 1 > m\angle 3$. Then $m\angle 2 > m\angle 3$ by substitution.



Got It? 1. Why is $m\angle 5 > m\angle C$?

You can use the corollary to Theorem 3-11 to prove the following theorem.

take note

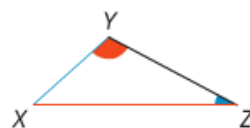
Theorem 5-10

Theorem

If two sides of a triangle are not congruent, then the larger angle lies opposite the longer side.

If . . .

$XZ > XY$



Then . . .

$m\angle Y > m\angle Z$

You will prove Theorem 5-10 in Exercise 40.

Think

How do you find the side opposite an angle?

Choose an angle of a triangle. The side opposite it is the only side that does not have an endpoint at the vertex of the angle.



Problem 2 Using Theorem 5-10

A town park is triangular. A landscape architect wants to place a bench at the corner with the largest angle. Which two streets form the corner with the largest angle?

Hollingsworth Road is the longest street, so it is opposite the largest angle. MLK Boulevard and Valley Road form the largest angle.



Got It? 2. Suppose the landscape architect wants to place a drinking fountain at the corner with the second largest angle. Which two streets form the corner with the second-largest angle?



Theorem 5-11 below is the converse of Theorem 5-10. The proof of Theorem 5-11 relies on indirect reasoning.



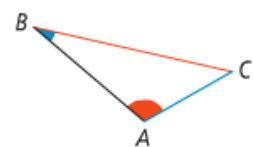
Theorem 5-11

Theorem

If two angles of a triangle are not congruent, then the longer side lies opposite the larger angle.

If ...

$$m\angle A > m\angle B$$



Then ...

$$BC > AC$$

Proof Indirect Proof of Theorem 5-11

Given: $m\angle A > m\angle B$

Prove: $BC > AC$

Step 1 Assume temporarily that $BC \not> AC$. That is, assume temporarily that either $BC < AC$ or $BC = AC$.

Step 2 If $BC < AC$, then $m\angle A < m\angle B$ (Theorem 5-10). This contradicts the given fact that $m\angle A > m\angle B$. Therefore, $BC < AC$ must be false.

If $BC = AC$, then $m\angle A = m\angle B$ (Isosceles Triangle Theorem). This also contradicts $m\angle A > m\angle B$. Therefore, $BC = AC$ must be false.

Step 3 The temporary assumption $BC \not> AC$ is false, so $BC > AC$.

Plan

How do you use the angle measures to order the side lengths?

List the angle measures in order from smallest to largest. Then replace the measure of each angle with the length of the side opposite.



Problem 3 Using Theorem 5-11

Multiple Choice Which choice shows the sides of $\triangle TUV$ in order from shortest to longest?

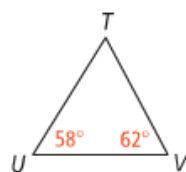
(A) $\overline{TV}, \overline{UV}, \overline{UT}$

(B) $\overline{UT}, \overline{UV}, \overline{TV}$

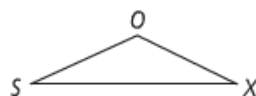
(C) $\overline{UV}, \overline{UT}, \overline{TV}$

(D) $\overline{TV}, \overline{UT}, \overline{UV}$

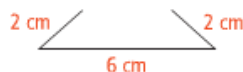
By the Triangle Angle-Sum Theorem, $m\angle T = 60$. $58 < 60 < 62$, so $m\angle U < m\angle T < m\angle V$. By Theorem 5-11, $TV < UV < UT$. Choice A is correct.



Got It? 3. **Reasoning** In the figure at the right, $m\angle S = 24$ and $m\angle O = 130$. Which side of $\triangle SOX$ is the shortest side? Explain your reasoning.



For three segments to form a triangle, their lengths must be related in a certain way. Notice that only one of the sets of segments below can form a triangle. The sum of the smallest two lengths must be greater than the greatest length.

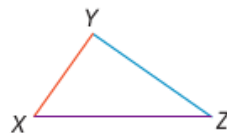


take note

Theorem 5-12 Triangle Inequality Theorem

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

$$XY + YZ > XZ \quad YZ + XZ > XY \quad XZ + XY > YZ$$



You will prove Theorem 5-12 in Exercise 45.

Think

How do you use the Triangle Inequality Theorem?

Test each pair of side lengths. The sum of each pair must be greater than the third length.



Problem 4 Using the Triangle Inequality Theorem

Can a triangle have sides with the given lengths? Explain.

A 3 ft, 7 ft, 8 ft

$$\begin{array}{lll} 3 + 7 > 8 & 7 + 8 > 3 & 8 + 3 > 7 \\ 10 > 8 & 15 > 3 & 11 > 7 \end{array}$$

Yes. The sum of the lengths of any two sides is greater than the length of the third side.

B 5 ft, 10 ft, 15 ft

No. The sum of 5 and 10 is not greater than 15. This contradicts Theorem 5-12.



Got It? 4. Can a triangle have sides with the given lengths? Explain.

a. 2 m, 6 m, and 9 m

b. 4 yd, 6 yd, and 9 yd



Problem 5 Finding Possible Side Lengths

Algebra In the Solve It, you explored the possible dimensions of a triangular sandbox. Two of the sides are 5 ft and 8 ft long. What is the range of possible lengths for the third side?

Know

The lengths of two sides of the triangle are 5 ft and 8 ft.

Need

The range of possible lengths of the third side

Plan

Use the Triangle Inequality Theorem to write three inequalities. Use the solutions of the inequalities to determine the greatest and least possible lengths.

Let x represent the length of the third side. Use the Triangle Inequality Theorem to write three inequalities. Then solve each inequality for x .

$$x + 5 > 8$$

$$x > 3$$

$$x + 8 > 5$$

$$x > -3$$

$$5 + 8 > x$$

$$x < 13$$

Numbers that satisfy $x > 3$ and $x < 13$ must be greater than 3. So, the third side must be greater than 3 ft and less than 13 ft.



Got It? 5. A triangle has side lengths of 4 in. and 7 in. What is the range of possible lengths for the third side?

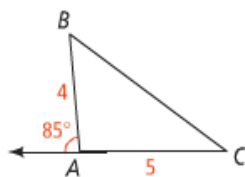


Lesson Check

Do you know HOW?

Use $\triangle ABC$ for Exercises 1 and 2.

- Which side is the longest?
- Which angle is the smallest?
- Can a triangle have sides of lengths 4, 5, and 10? Explain.



Do you UNDERSTAND?

- Error Analysis** A friend tells you that she drew a triangle with perimeter 16 and one side of length 8. How do you know she made an error in her drawing?
- Reasoning** Is it possible to draw a right triangle with an exterior angle measuring 88? Explain your reasoning.



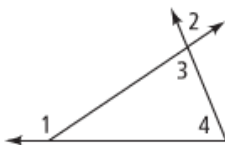
Practice and Problem-Solving Exercises



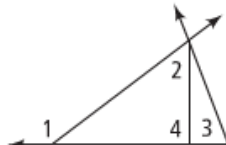
Practice

Explain why $m\angle 1 > m\angle 2$.

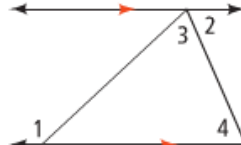
6.



7.



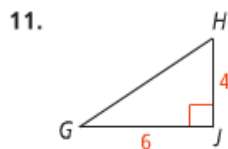
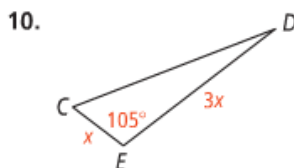
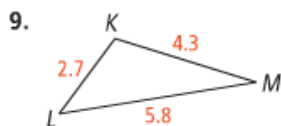
8.



See Problem 1.

For Exercises 9–14, list the angles of each triangle in order from smallest to largest.

See Problem 2.



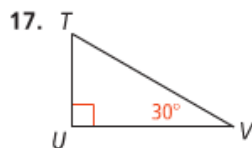
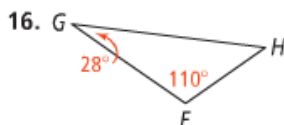
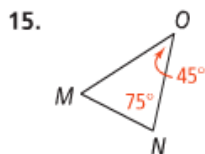
12. $\triangle ABC$, where $AB = 8$, $BC = 5$, and $CA = 7$

13. $\triangle DEF$, where $DE = 15$, $EF = 18$, and $DF = 5$

14. $\triangle XYZ$, where $XY = 12$, $YZ = 24$, and $ZX = 30$

For Exercises 15–20, list the sides of each triangle in order from shortest to longest.

See Problem 3.



18. $\triangle ABC$, with $m\angle A = 90$, $m\angle B = 40$, and $m\angle C = 50$

19. $\triangle DEF$, with $m\angle D = 20$, $m\angle E = 120$, and $m\angle F = 40$

20. $\triangle XYZ$, with $m\angle X = 51$, $m\angle Y = 59$, and $m\angle Z = 70$

Can a triangle have sides with the given lengths? Explain.

See Problem 4.

21. 2 in., 3 in., 6 in.

22. 11 cm, 12 cm, 15 cm

23. 8 m, 10 m, 19 m

24. 1 cm, 15 cm, 15 cm

25. 2 yd, 9 yd, 10 yd

26. 4 m, 5 m, 9 m

Algebra The lengths of two sides of a triangle are given. Find the range of possible lengths for the third side.

See Problem 5.

27. 8 ft, 12 ft

28. 5 in., 16 in.

29. 6 cm, 6 cm

30. 18 m, 23 m

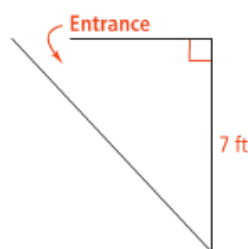
31. 4 yd, 7 yd

32. 20 km, 35 km



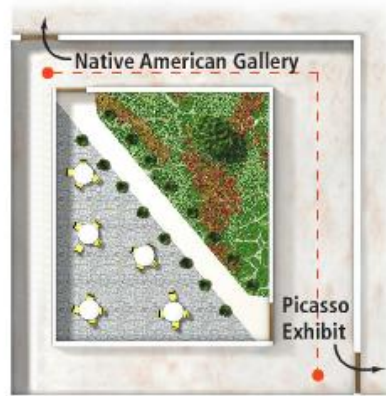
33. **Think About a Plan** You are setting up a study area where you will do your homework each evening. It is triangular with an entrance on one side. You want to put your computer in the corner with the largest angle and a bookshelf on the longest side. Where should you place your computer? On which side should you place the bookshelf? Explain.

- What type of triangle is shown in the figure?
- Once you find the largest angle of a triangle, how do you find the longest side?



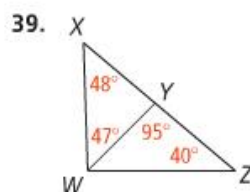
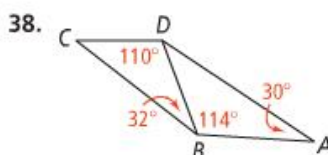
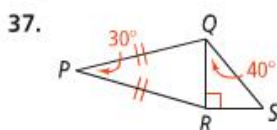
34. **Algebra** Find the longest side of $\triangle ABC$, with $m\angle A = 70$, $m\angle B = 2x - 10$, and $m\angle C = 3x + 20$.

35. **Writing** You and a friend compete in a scavenger hunt at a museum. The two of you walk from the Picasso exhibit to the Native American gallery along the dashed red line. When he sees that another team is ahead of you, your friend says, "They must have cut through the courtyard." Explain what your friend means.



36. **Error Analysis** Your family drives across Kansas on Interstate 70. A sign reads, "Wichita 90 mi, Topeka 110 mi." Your little brother says, "I didn't know that it was only 20 miles from Wichita to Topeka." Explain why the distance between the two cities does not have to be 20 mi.

Reasoning Determine which segment is shortest in each diagram.



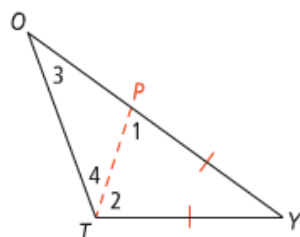
40. **Developing Proof** Fill in the blanks for a proof of Theorem 5-10: If two sides of a triangle are not congruent, then the larger angle lies opposite the longer side.

Given: $\triangle TOY$, with $YO > YT$

Prove: a. $\angle Y > \angle T$

Mark P on $\overline{YO}$ so that $\overline{YP} \cong \overline{YT}$. Draw $\overline{TP}$.

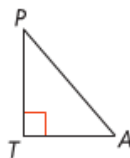
Statements	Reasons
1) $\overline{YP} \cong \overline{YT}$	1) Ruler Postulate
2) $m\angle 1 = m\angle 2$	2) c. $\angle$
3) $m\angle OTY = m\angle 4 + m\angle 2$	3) d. $\angle$
4) $m\angle OTY > m\angle 2$	4) e. $\angle$
5) $m\angle OTY > m\angle 1$	5) f. $\angle$
6) $m\angle 1 > m\angle 3$	6) g. $\angle$
7) $m\angle OTY > m\angle 3$	7) h. $\angle$



41. **Proof** Prove this corollary to Theorem 5-11: The perpendicular segment from a point to a line is the shortest segment from the point to the line.

Given: $\overline{PT} \perp \overline{TA}$

Prove: $PA > PT$



42. **Probability** A student has two straws. One is 6 cm long and the other is 9 cm long. She picks a third straw at random from a group of four straws whose lengths are 3 cm, 5 cm, 11 cm, and 15 cm. What is the probability that the straw she picks will allow her to form a triangle? Justify your answer.

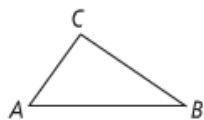
For Exercises 43 and 44, x and y are integers such that $1 < x < 5$ and $2 < y < 9$.

43. The sides of a triangle are 5 cm, x cm, and y cm. List all possible (x, y) pairs.
44. **Probability** What is the probability that you can draw an isosceles triangle that has sides 5 cm, x cm, and y cm, with x and y chosen at random?
45. **Proof** Prove the Triangle Inequality Theorem: The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Given: $\triangle ABC$

Prove: $AC + CB > AB$

(Hint: On $\overrightarrow{BC}$, mark a point D not on $\overline{BC}$, so that $DC = AC$. Draw $\overline{DA}$ and use Theorem 5-11 with $\triangle ABD$.)

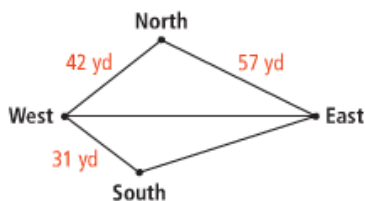


Standardized Test Prep

GRIDDED RESPONSE



46. The figure shows the walkways connecting four dormitories on a college campus. What is the greatest possible whole-number length, in yards, for the walkway between South dorm and East dorm?
47. What is the length of a segment with endpoints $A(-13, -16)$ and $B(-9, -13)$?
48. How many sides does a convex quadrilateral have?
49. $\angle 1$ and $\angle 2$ are corresponding angles formed by two parallel lines and a transversal. If $m\angle 1 = 33x + 2$ and $m\angle 2 = 68$, what is the value of x ?



Mixed Review

Write the first step of an indirect proof of the given statement.

50. The side is at least 2 ft long.
51. $\triangle PQR$ has two congruent angles.
52. You know that $\overline{AB} \cong \overline{XY}$, $\overline{BC} \cong \overline{YZ}$, and $\overline{CA} \cong \overline{ZX}$. By what theorem or postulate can you conclude that $\triangle ABC \cong \triangle XYZ$?

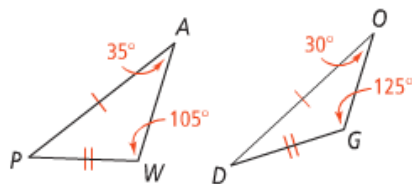
See Lesson 5-5.

See Lesson 4-2.

Get Ready! To prepare for Lesson 5-7, do Exercises 53-55.

Use the figure at the right.

53. What is $m\angle P$?
54. What is $m\angle D$?
55. Is it possible for AW to equal OG ?



See Lesson 3-5.

Inequalities in Two Triangles

Objective To apply inequalities in two triangles



You can compare distances without using a ruler.



Getting Ready!

Think of a clock or watch that has an hour hand and a minute hand. As minutes pass, the distance between the tip of the hour hand and the tip of the minute hand changes. This distance is x in the figure at the right. What is the order of the times below from least to greatest length of x ? How do you know?

1:00, 3:00, 5:00, 8:30, 1:30, 12:20



In the Solve It, the hands of the clock and the segment labeled x form a triangle. As the time changes, the shape of the triangle changes, but the lengths of two of its sides do not change.

Essential Understanding In triangles that have two pairs of congruent sides, there is a relationship between the included angles and the third pair of sides.

When you close a door, the angle between the door and the frame (at the hinge) gets smaller. The relationship between the measure of the hinge angle and the length of the opposite side is the basis for the SAS Inequality Theorem, also known as the Hinge Theorem.



take note

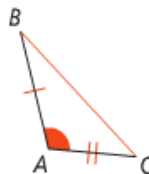
Theorem 5-13 The Hinge Theorem (SAS Inequality Theorem)

Theorem

If two sides of one triangle are congruent to two sides of another triangle, and the included angles are not congruent, then the longer third side is opposite the larger included angle.

If ...

$$m\angle A > m\angle X$$



Then ...

$$BC > YZ$$

You will prove Theorem 5-13 in Exercise 25.

Plan

How do you apply the Hinge Theorem?

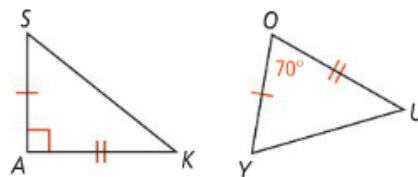
After you identify the angles included between the pairs of congruent sides, locate the sides opposite those angles.



Problem 1 Using the Hinge Theorem

Multiple Choice Which of the following statements must be true?

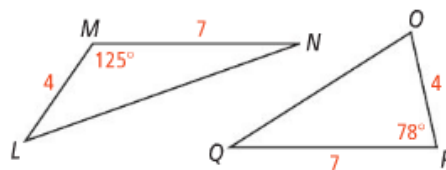
- (A) $AS < YU$ (C) $SK < YU$
 (B) $SK > YU$ (D) $AK = YU$



$\overline{SA} \cong \overline{YO}$ and $\overline{AK} \cong \overline{OU}$, so the triangles have two pairs of congruent sides. The included angles, $\angle A$ and $\angle O$, are not congruent. Since $m\angle A > m\angle O$, $SK > YU$ by the Hinge Theorem. The correct answer is B.



- Got It?** 1. a. What inequality relates LN and OQ in the figure at the right?
 b. **Reasoning** In $\triangle ABC$, $AB = 3$, $BC = 4$, and $CA = 6$. In $\triangle PQR$, $PQ = 3$, $QR = 5$, and $RP = 6$. How can you use indirect reasoning to explain why $m\angle P > m\angle A$?



Problem 2 Applying the Hinge Theorem

Swing Ride The diagram below shows the position of a swing at two different times. As the speed of the swing ride increases, the angle between the chain and $\overline{AB}$ increases. Is the rider farther from point A at Time 1 or Time 2? Explain how the Hinge Theorem justifies your answer.

The rider is farther from point A at Time 2. The lengths of $\overline{AB}$ and $\overline{BC}$ stay the same throughout the ride. Since the angle formed at Time 2 ($\angle 2$) is greater than the angle formed at Time 1 ($\angle 1$), you can use the Hinge Theorem to conclude that $\overline{AC}$ at Time 2 is longer than $\overline{AC}$ at Time 1.



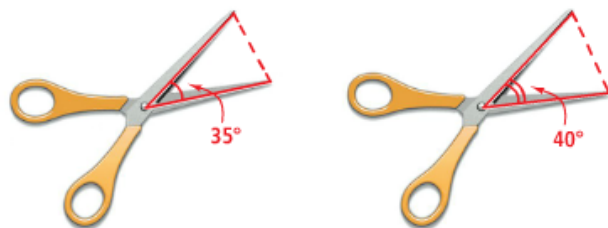
Think

For $\triangle ABC$, which side lengths are the same at Time 1 and Time 2?

The lengths of the chain and $\overline{AB}$ do not change. So, AB and BC are the same at Time 1 and Time 2.



- Got It?** 2. The diagram below shows a pair of scissors in two different positions. In which position is the distance between the tips of the two blades greater? Use the Hinge Theorem to justify your answer.



The Converse of the Hinge Theorem is also true. The proof of the converse is an indirect proof.

Take note

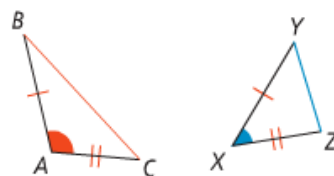
Theorem 5-14 Converse of the Hinge Theorem (SSS Inequality)

Theorem

If two sides of one triangle are congruent to two sides of another triangle, and the third sides are not congruent, then the larger included angle is opposite the longer third side.

If ...

$$BC > YZ$$



Then ...

$$m\angle A > m\angle X$$

Proof Indirect Proof of the Converse of the Hinge Theorem (SSS Inequality)

Given: $\overline{AB} \cong \overline{XY}$, $\overline{AC} \cong \overline{XZ}$,
 $BC > YZ$

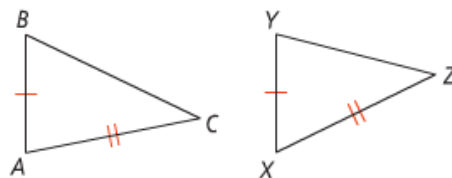
Prove: $m\angle A > m\angle X$

Step 1 Assume temporarily that $m\angle A \not> m\angle X$.
This means either $m\angle A < m\angle X$ or
 $m\angle A = m\angle X$.

Step 2 If $m\angle A < m\angle X$, then $BC < YZ$ by the Hinge Theorem. This contradicts the given information that $BC > YZ$. Therefore, the assumption that $m\angle A < m\angle X$ must be false.

If $m\angle A = m\angle X$, then $\triangle ABC \cong \triangle XYZ$ by SAS. If the two triangles are congruent, then $BC = YZ$ because corresponding parts of congruent triangles are congruent. This contradicts the given information that $BC > YZ$. Therefore, the assumption that $m\angle A = m\angle X$ must be false.

Step 3 The temporary assumption that $m\angle A \not> m\angle X$ is false. Therefore,
 $m\angle A > m\angle X$.



**Problem 3 Using the Converse of the Hinge Theorem****Algebra** What is the range of possible values for x ?

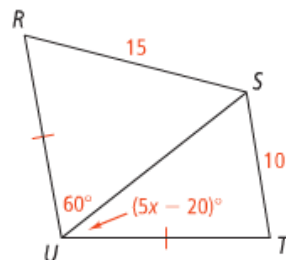
Step 1 Find an upper limit for the value of x . $\overline{UT} \cong \overline{UR}$ and $\overline{US} \cong \overline{US}$, so $\triangle TUS$ and $\triangle RUS$ have two pairs of congruent sides. $RS > TS$, so you can use the Converse of the Hinge Theorem to write an inequality.

$$m\angle RUS > m\angle TUS \quad \text{Converse of the Hinge Theorem}$$

$$60 > 5x - 20 \quad \text{Substitute.}$$

$$80 > 5x \quad \text{Add 20 to each side.}$$

$$16 > x \quad \text{Divide each side by 5.}$$



Step 2 Find a lower limit for the value of x .

$$m\angle TUS > 0 \quad \text{The measure of an angle of a triangle is greater than 0.}$$

$$5x - 20 > 0 \quad \text{Substitute.}$$

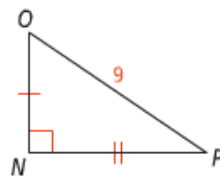
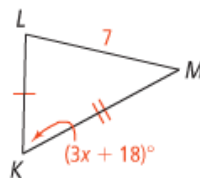
$$5x > 20 \quad \text{Add 20 to each side.}$$

$$x > 4 \quad \text{Divide each side by 5.}$$

Rewrite $16 > x$ and $x > 4$ as $4 < x < 16$.

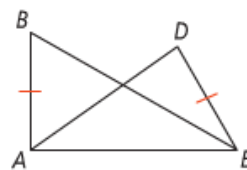


Got It? 3. What is the range of possible values for x in the figure at the right?

**Proof****Problem 4 Proving Relationships in Triangles**

Given: $BA = DE$, $BE > DA$

Prove: $m\angle BAE > m\angle BEA$

**Statement****Reasons**

1) $BA = DE$

1) Given

2) $AE = AE$

2) Reflexive Property of Equality

3) $BE > DA$

3) Given

4) $m\angle BAE > m\angle DEA$

4) Converse of the Hinge Theorem

5) $m\angle DEA = m\angle DEB + m\angle BEA$

5) Angle Addition Postulate

6) $m\angle DEA > m\angle BEA$

6) Comparison Property of Inequality

7) $m\angle BAE > m\angle BEA$

7) Transitive Property of Inequality



Got It? 4. **Given:** $m\angle MON = 80$, O is the midpoint of $\overline{LN}$

Prove: $LM > MN$

**Plan**

How do you put upper and lower limits on the value of x ?

Use the largest possible value of $m\angle TUS$ as the upper limit for $5x - 20$ and the smallest possible value of $m\angle TUS$ as the lower limit for $5x - 20$.

Think

How do you know $m\angle BAE > m\angle BEA$? Use the Transitive Property of Inequality on the inequalities in Statements 4 and 6.

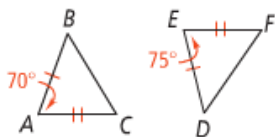


Lesson Check

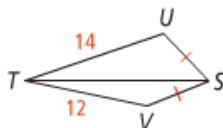
Do you know HOW?

Write an inequality relating the given side lengths or angle measures.

1. FD and BC



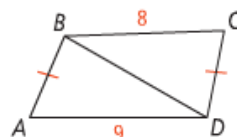
2. $m\angle UST$ and $m\angle VST$



Do you UNDERSTAND?

3. **Vocabulary** Explain why *Hinge Theorem* is an appropriate name for Theorem 5-13.

4. **Error Analysis** From the figure at the right, your friend concludes that $m\angle BAD > m\angle BCD$. How would you correct your friend's mistake?



5. **Compare and Contrast** How are the Hinge Theorem and the SAS Congruence Postulate similar?



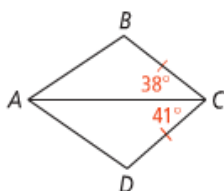
Practice and Problem-Solving Exercises

A Practice

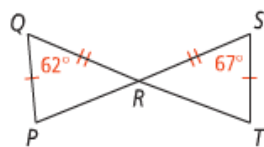
Write an inequality relating the given side lengths. If there is not enough information to reach a conclusion, write *no conclusion*.

See Problem 1.

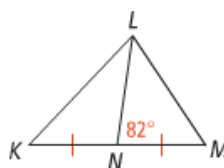
6. AB and AD



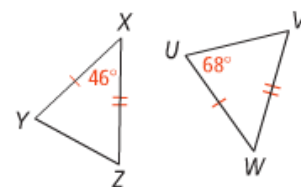
7. PR and RT



8. LM and KL

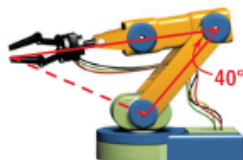
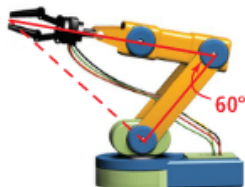


9. YZ and UV



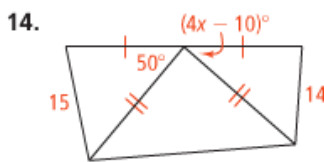
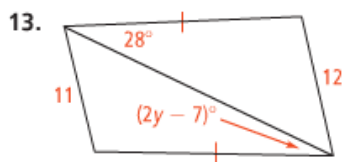
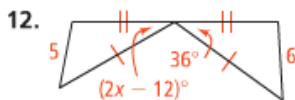
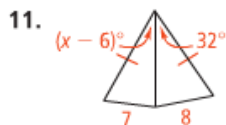
10. The diagram below shows a robotic arm in two different positions. In which position is the tip of the robotic arm closer to the base? Use the Hinge Theorem to justify your answer.

See Problem 2.



Algebra Find the range of possible values for each variable.

See Problem 3.



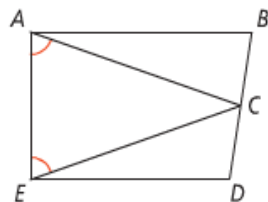
15. **Developing Proof** Complete the following proof.

See Problem 4.

Given: C is the midpoint of $\overline{BD}$,
 $m\angle EAC = m\angle AEC$,
 $m\angle BCA > m\angle DCE$

Prove: $AB > ED$

Statements	Reasons
1) $m\angle EAC = m\angle AEC$	1) Given
2) $AC = EC$	2) a. ?
3) C is the midpoint of $\overline{BD}$.	3) b. ?
4) $\overline{BC} \cong \overline{CD}$	4) c. ?
5) d. ?	5) $\cong$ segments have = length.
6) $m\angle BCA > m\angle DCE$	6) e. ?
7) $AB > ED$	7) f. ?



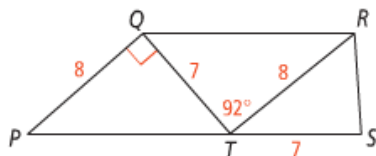
B Apply

Copy and complete with $>$ or $<$. Explain your reasoning.

16. $PT \square QR$

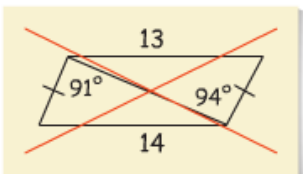
17. $m\angle QTR \square m\angle RTS$

18. $PT \square RS$



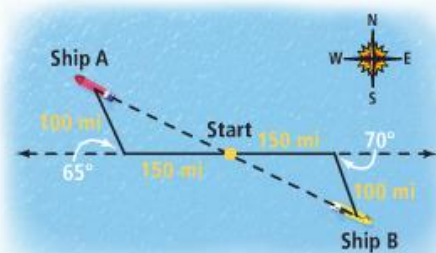
19. **a. Error Analysis** Your classmate draws the figure at the right. Explain why the figure cannot have the labeled dimensions.

b. Open-Ended Describe a way you could change the dimensions to make the figure possible.



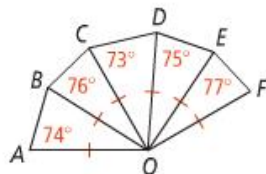
20. **Think About a Plan** Ship A and Ship B leave from the same point in the ocean. Ship A travels 150 mi due west, turns 65° toward north, and then travels another 100 mi. Ship B travels 150 mi due east, turns 70° toward south, and then travels another 100 mi. Which ship is farther from the starting point? Explain.

- How can you use the given angle measures?
- How does the Hinge Theorem help you to solve this problem?



21. Which of the following lists the segment lengths in order from least to greatest?

- (A) CD, AB, DE, BC, EF
 (B) EF, DE, AB, BC, CD
 (C) BC, DE, EF, AB, CD
 (D) EF, BC, DE, AB, CD

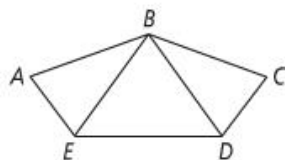


22. **Reasoning** The legs of a right isosceles triangle are congruent to the legs of an isosceles triangle with an 80° vertex angle. Which triangle has a greater perimeter? How do you know?

23. Use the figure at the right.

Given: $\triangle ABE$ is isosceles with vertex $\angle B$,
 $\triangle ABE \cong \triangle CBD$,
 $m\angle EBD > m\angle ABE$

Prove: $ED > AE$



24. **Coordinate Geometry** $\triangle ABC$ has vertices $A(0, 7)$, $B(-1, -2)$, $C(2, -1)$, and $O(0, 0)$. Show that $m\angle AOB > m\angle AOC$.

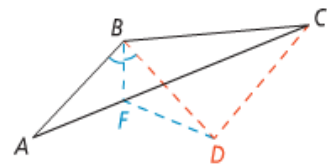
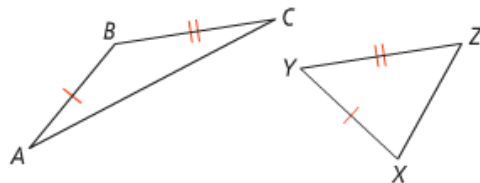
25. **Proof** Use the plan below to complete a proof of the Hinge Theorem: If two sides of one triangle are congruent to two sides of another triangle and the included angles are not congruent, then the longer third side is opposite the larger included angle.

Given: $\overline{AB} \cong \overline{XY}$, $\overline{BC} \cong \overline{YZ}$, $m\angle B > m\angle Y$

Prove: $AC > XZ$

Plan for proof:

- Copy $\triangle ABC$. Locate point D outside $\triangle ABC$ so that $m\angle CBD = m\angle ZYX$ and $BD = YX$. Show that $\triangle DBC \cong \triangle XYZ$.
- Locate point F on $\overline{AC}$, so that $\overline{BF}$ bisects $\angle ABD$.
- Show that $\triangle ABF \cong \triangle DBF$ and that $\overline{AF} \cong \overline{DF}$.
- Show that $AC = FC + DF$.
- Use the Triangle Inequality Theorem to write an inequality that relates DC to the lengths of the other sides of $\triangle FCD$.
- Relate DC and XZ .



Standardized Test Prep

SAT/ACT

26. Which is a possible value for x in the figure at the right?

(A) 2

(C) 6

(B) 4

(D) 8

27. A quilter cuts out and sews together two triangles, as shown in the figure. She tries to make $\triangle ABD$ and $\triangle BDC$ equilateral triangles, but the angle measures are as shown. Assuming that the side $\overline{BD}$ is actually the same length in both triangles, which segment is the longest side in the figure?

(F) $\overline{AB}$

(H) $\overline{CD}$

(G) $\overline{BC}$

(I) $\overline{AD}$

28. What is the equation of the perpendicular bisector of $\overline{MN}$ with endpoints $M(8, 0)$ and $N(0, 0)$?

(A) $y = x - 8$

(C) $x = 4$

(B) $x = 8$

(D) $y = 4$

29. The orthocenter of a triangle lies outside the triangle. Which of the following statements cannot be true?

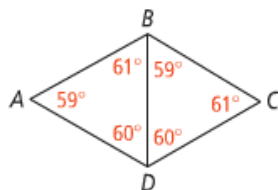
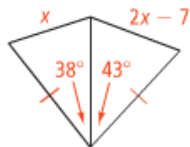
(F) The triangle has a 120° angle.

(H) The incenter is inside the triangle.

(G) The triangle is isosceles.

(I) The triangle is acute.

30. Use indirect reasoning to write a convincing argument that a triangle has at most one obtuse angle.

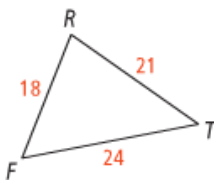


Short
Response

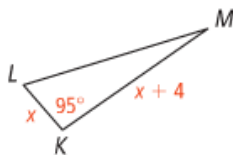
Mixed Review

List the angles of each triangle in order from smallest to largest.

31.



32.



See Lesson 5-6.

Algebra The lengths of two sides of a triangle are given. Find the range of possible lengths for the third side.

33. 15 cm, 19 cm

34. 6 ft, 11 ft

35. 3 in., 3 in.

36. In $\triangle GHI$, which side is included between $\angle G$ and $\angle H$?

See Lesson 4-2.

Get Ready! To prepare for Lesson 6-1, do Exercises 37-39.

Find the slope of the line through each pair of points.

See Lesson 3-7.

37. $X(0, 6)$, $Y(4, 9)$

38. $R(3, 8)$, $S(6, 0)$

39. $A(4, 3)$, $B(2, 1)$

Pull It All Together

To solve these problems you will pull together many concepts and skills that you have studied about relationships within triangles.



BIG idea Coordinate Geometry

You can use the Midpoint Formula, the slope formula, and the relationship between perpendicular lines to find points of concurrency.

Task 1

Your math teacher manages a campground during summer vacation. He loves math so much that he has mapped the campground on a coordinate grid. The campsites have the following coordinates: Brighton Bluff at $B(2, 2)$, Ponaganset Peak at $P(4, 10)$, and Harmony Hill at $H(12, 2)$. He wants to build showers that are equidistant from all three campsites. Find the coordinates of the point where the showers should be placed.

BIG idea Reasoning and Proof

You can use proven theorems to explore relationships among sides, angles, and special lines and segments in triangles.

Task 2

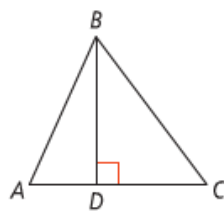
- Draw $\triangle ABC$ with obtuse $\angle C$ and construct its orthocenter O . Then find the orthocenters of $\triangle ABO$, $\triangle ACO$, and $\triangle BCO$. What did you discover? Explain why you get this result.
- Will your conjecture be true for any acute or right $\triangle ABC$? Explain your reasoning.

BIG idea Reasoning and Proof

You can use indirect reasoning to prove relationships within triangles.

Task 3

In $\triangle ABC$, $AB \neq BC$. Show that there does not exist a point P on altitude $\overline{BD}$ that is equidistant from A and C .



Connecting **BIG** ideas and Answering the Essential Questions**1 Coordinate Geometry**

Use parallel and perpendicular lines, and the slope, midpoint, and distance formulas to find intersection points and unknown lengths.

2 Measurement

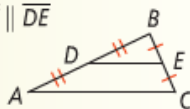
Use theorems about perpendicular bisectors, angle bisectors, medians, and altitudes to find points of concurrency, angle measures, and segment lengths.

3 Reasoning and Proof

You can write an indirect proof by showing that a temporary assumption is false.

Midsegments of Triangles (Lesson 5-1)

If $\overline{DE}$ is a midsegment, then $\overline{AC} \parallel \overline{DE}$ and $DE = \frac{1}{2}AC$.

**Concurrent Lines and Segments in Triangles (Lessons 5-2, 5-3, and 5-4)****Concurrent Lines and Segments**

- perpendicular bisectors
- angle bisectors
- medians
- lines containing altitudes

Intersection

- circumcenter
- incenter
- centroid
- orthocenter

Indirect Proof (Lesson 5-5)

- 1) Assume temporarily the opposite of what you want to prove.
- 2) Show that this temporary assumption leads to a contradiction.
- 3) Conclude that what you want to prove is true.

Inequalities in Triangles (Lessons 5-6 and 5-7)

Use indirect reasoning to prove that the longer of two sides of a triangle lies opposite the larger angle, and to prove the Converse of the Hinge Theorem.

**Chapter Vocabulary**

- altitude of a triangle (p. 310)
- centroid of a triangle (p. 309)
- circumcenter of a triangle (p. 301)
- circumscribed about (p. 301)
- concurrent (p. 301)
- distance from a point to a line (p. 294)
- equidistant (p. 292)
- incenter of a triangle (p. 303)
- indirect proof (p. 317)
- indirect reasoning (p. 317)
- inscribed in (p. 303)
- median of a triangle (p. 309)
- midsegment of a triangle (p. 285)
- orthocenter of a triangle (p. 311)
- point of concurrency (p. 301)

Choose the correct vocabulary term to complete each sentence.

1. A (*centroid, median*) of a triangle is a segment from a vertex of the triangle to the midpoint of the side opposite the vertex.
2. The length of the perpendicular segment from a point to a line is the (*midsegment, distance from a point to the line*).
3. The (*circumcenter, incenter*) of a triangle is the point of concurrency of the angle bisectors of the triangle.

5-1 Midsegments of Triangles

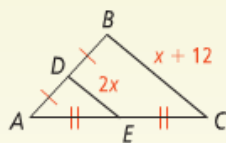
Quick Review

A **midsegment of a triangle** is a segment that connects the midpoints of two sides. A midsegment is parallel to the third side and is half as long.

Example

Algebra Find the value of x .

$\overline{DE}$ is a midsegment because D and E are midpoints.



$$DE = \frac{1}{2}BC \quad \triangle \text{ Midsegment Theorem}$$

$$2x = \frac{1}{2}(x + 12) \quad \text{Substitute.}$$

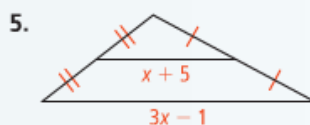
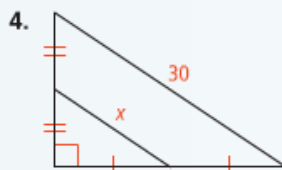
$$4x = x + 12 \quad \text{Simplify.}$$

$$3x = 12 \quad \text{Subtract } x \text{ from each side.}$$

$$x = 4 \quad \text{Divide each side by 3.}$$

Exercises

Algebra Find the value of x .



6. $\triangle ABC$ has vertices $A(0, 0)$, $B(2, 2)$, and $C(5, -1)$. Find the coordinates of L , the midpoint of $\overline{AC}$, and M , the midpoint of $\overline{BC}$. Verify that $\overline{LM} \parallel \overline{AB}$ and $LM = \frac{1}{2}AB$.

5-2 Perpendicular and Angle Bisectors

Quick Review

The **Perpendicular Bisector Theorem** together with its converse states that P is equidistant from A and B if and only if P is on the perpendicular bisector of $\overline{AB}$.

The **distance from a point to a line** is the length of the perpendicular segment from the point to the line.

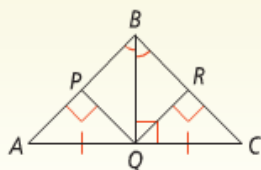
The **Angle Bisector Theorem** together with its converse states that P is equidistant from the sides of an angle if and only if P is on the angle bisector.

Example

In the figure, $QP = 4$ and $AB = 8$. Find QR and CB .

Q is on the bisector of $\angle ABC$, so $QR = QP = 4$.

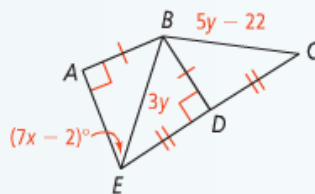
B is on the perpendicular bisector of $\overline{AC}$, so $CB = AB = 8$.



Exercises

7. **Writing** Describe how to find all the points on a baseball field that are equidistant from second base and third base.

In the figure, $m\angle DBE = 50$. Find each of the following.



- | | |
|------------------|------------------|
| 8. $m\angle BED$ | 9. $m\angle BEA$ |
| 10. x | 11. y |
| 12. BE | 13. BC |

5-3 Bisectors in Triangles

Quick Review

When three or more lines intersect in one point, they are **concurrent**.

- The point of concurrency of the perpendicular bisectors of a triangle is the **circumcenter of the triangle**.
- The point of concurrency of the angle bisectors of a triangle is the **incenter of the triangle**.

Example

Identify the incenter of the triangle.

The incenter of a triangle is the point of concurrency of the angle bisectors. $\overline{MR}$ and $\overline{LQ}$ are angle bisectors that intersect at Z . So, Z is the incenter.



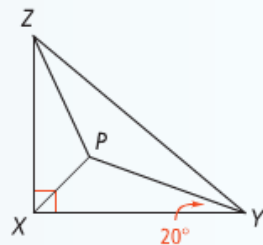
Exercises

Find the coordinates of the circumcenter of $\triangle DEF$.

- $D(6, 0), E(0, 6), F(-6, 0)$
- $D(0, 0), E(6, 0), F(0, 4)$
- $D(5, -1), E(-1, 3), F(3, -1)$
- $D(2, 3), E(8, 3), F(8, -1)$

P is the incenter of $\triangle XYZ$. Find the indicated angle measure.

- $m\angle PXY$
- $m\angle XYZ$
- $m\angle PZX$



5-4 Medians and Altitudes

Quick Review

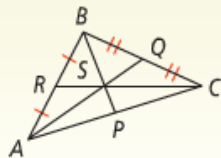
A **median of a triangle** is a segment from a vertex to the midpoint of the opposite side. An **altitude of a triangle** is a perpendicular segment from a vertex to the line containing the opposite side.

- The point of concurrency of the medians of a triangle is the **centroid of the triangle**. The centroid is two thirds the distance from each vertex to the midpoint of the opposite side.
- The point of concurrency of the altitudes of a triangle is the **orthocenter of the triangle**.

Example

If $PB = 6$, what is SB ?

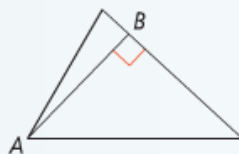
S is the centroid because $\overline{AQ}$ and $\overline{CR}$ are medians. So, $SB = \frac{2}{3}PB = \frac{2}{3}(6) = 4$.



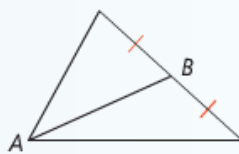
Exercises

Determine whether $\overline{AB}$ is a *median*, an *altitude*, or *neither*. Explain.

21.



22.



23. $\triangle PQR$ has medians $\overline{QM}$ and $\overline{PN}$ that intersect at Z . If $ZM = 4$, find QZ and QM .

$\triangle ABC$ has vertices $A(2, 3)$, $B(-4, -3)$, and $C(2, -3)$. Find the coordinates of each point of concurrency.

- centroid
- orthocenter

5-5 Indirect Proof

Quick Review

In an **indirect proof**, you first assume temporarily the opposite of what you want to prove. Then you show that this temporary assumption leads to a contradiction.

Example

Which two statements contradict each other?

- I. The perimeter of $\triangle ABC$ is 14.
- II. $\triangle ABC$ is isosceles.
- III. The side lengths of $\triangle ABC$ are 3, 5, and 6.

An isosceles triangle can have a perimeter of 14.

The perimeter of a triangle with side lengths 3, 5, and 6 is 14.

An isosceles triangle must have two sides of equal length.

Statements II and III contradict each other.

Exercises

Write a convincing argument that uses indirect reasoning.

26. The product of two numbers is even. Show that at least one of the numbers must be even.
27. Two lines in the same plane are not parallel. Show that a third line in the plane must intersect at least one of the two lines.
28. Show that a triangle can have at most one obtuse angle.
29. Show that an equilateral triangle cannot have an obtuse angle.
30. The sum of three integers is greater than 9. Show that one of the integers must be greater than 3.

5-6 and 5-7 Inequalities in Triangles

Quick Review

For any triangle,

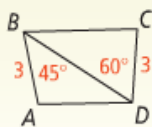
- the measure of an exterior angle is greater than the measure of each of its remote interior angles
- if two sides are not congruent, then the larger angle lies opposite the longer side
- if two angles are not congruent, then the longer side lies opposite the larger angle
- the sum of any two side lengths is greater than the third

The **Hinge Theorem** states that if two sides of one triangle are congruent to two sides of another triangle, and the included angles are not congruent, then the longer third side is opposite the larger included angle.

Example

Which is greater, BC or AD ?

$\overline{BA} \cong \overline{CD}$ and $\overline{BD} \cong \overline{DB}$, so $\triangle ABD$ and $\triangle CBD$ have two pairs of congruent corresponding sides. Since $60^\circ > 45^\circ$, you know $BC > AD$ by the Hinge Theorem.



Exercises

31. In $\triangle RST$, $m\angle R = 70$ and $m\angle S = 80$. List the sides of $\triangle RST$ in order from shortest to longest.

Is it possible for a triangle to have sides with the given lengths? Explain.

32. 5 in., 8 in., 15 in.

33. 10 cm, 12 cm, 20 cm

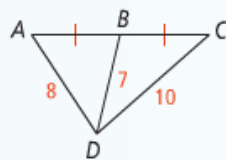
34. The lengths of two sides of a triangle are 12 ft and 13 ft. Find the range of possible lengths for the third side.

Use the figure below. Complete each statement with $>$, $<$, or $=$.

35. $m\angle BAD$ $\square$ $m\angle ABD$

36. $m\angle CBD$ $\square$ $m\angle BCD$

37. $m\angle ABD$ $\square$ $m\angle CBD$



**Do you know HOW?**Find the coordinates of the circumcenter of $\triangle ABC$.

1. $A(3, -1), B(-2, -1), C(3, -8)$
2. $A(0, 5), B(-4, 5), C(-4, -3)$

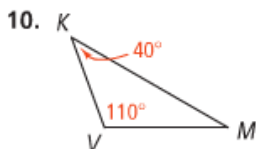
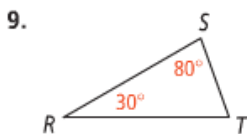
Find the coordinates of the orthocenter of $\triangle ABC$.

3. $A(-1, -1), B(-1, 5), C(-4, -1)$
4. $A(0, 0), B(5, 0), C(5, 3)$

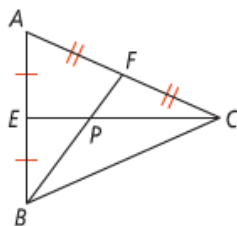
Identify the two statements that contradict each other.

5. I. $\triangle PQR$ is a right triangle.
II. $\triangle PQR$ is an obtuse triangle.
III. $\triangle PQR$ is scalene.
6. I. $\angle DOS \cong \angle CAT$
II. $\angle DOS$ and $\angle CAT$ are vertical.
III. $\angle DOS$ and $\angle CAT$ are adjacent.
7. If $AB = 9, BC = 4\frac{1}{2}$, and $AC = 12$, list the angles of $\triangle ABC$ from smallest to largest.
8. Point P is inside $\triangle ABC$ and equidistant from all three sides. If $m\angle ABC = 60$, what is $m\angle PBC$?

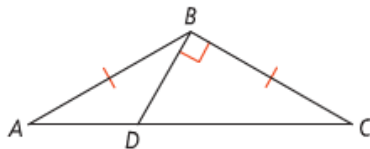
List the sides from shortest to longest.



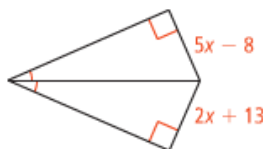
11. In $\triangle ABC$, $EP = 4$.
What is PC ?



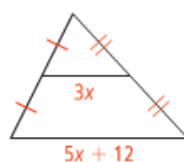
12. Which is greater, AD or DC ? Explain.

**Algebra** Find the value of x .

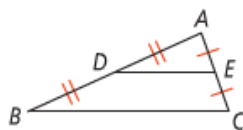
13.



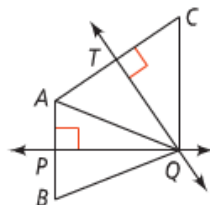
14.

**Do you UNDERSTAND?**

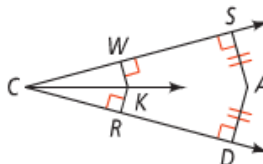
15. What can you conclude from the diagram at the right? Justify your answer.
16. **Reasoning** In $\triangle ABC$, $BC > BA$. Draw $\triangle ABC$ and the median $\overline{BD}$. Use the Converse of the Hinge Theorem to explain why $\angle BDC$ is obtuse.



17. **Given:** $\overleftrightarrow{PQ}$ is the perpendicular bisector of $\overline{AB}$. $\overleftrightarrow{QT}$ is the perpendicular bisector of $\overline{AC}$.

Prove: $QC = QB$ 

18. **Writing** Use indirect reasoning to explain why the following statement is true: If an isosceles triangle is obtuse, then the obtuse angle is the vertex angle.
19. In the figure, $WK = KR$. What can you conclude about point A ? Explain.



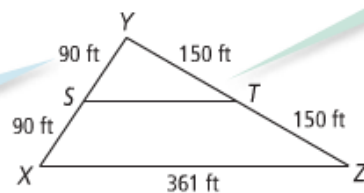
TIPS FOR SUCCESS

Some test questions ask you to find missing measurements. Read the sample question at the right. Then follow the tips to answer it.

TIP 2

From the dimensions given in the diagram, you can conclude that S and T are midpoints of sides of $\triangle XYZ$. So you can use the Triangle Midsegment Theorem to find ST .

The diagram below shows the walkways in a triangular park. What is the distance from point S to point Z using the walkways $\overline{ST}$ and $\overline{TZ}$?



- (A) 180.5 ft (C) 390 ft
(B) 330.5 ft (D) 451 ft

TIP 1

You need to find the length of $\overline{ST}$ and add it to 150 ft, the length of $\overline{TZ}$.

Think It Through

$\overline{ST}$ is a midsegment of $\triangle XYZ$. By the Triangle Midsegment Theorem, you know $ST = \frac{1}{2}(XZ) = \frac{1}{2}(361)$, or 180.5 ft. The distance from point S to point Z using walkways $\overline{ST}$ and $\overline{TZ}$ is $ST + TZ = 180.5 + 150$, or 330.5 ft. The correct answer is B.



Vocabulary Builder

As you solve test items, you must understand the meanings of mathematical terms. Choose the correct term to complete each sentence.

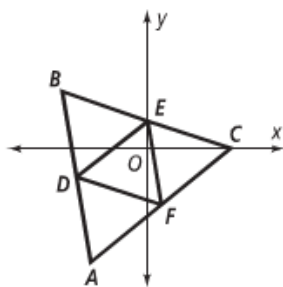
- I. The (*inverse*, *converse*) of a conditional statement negates both the hypothesis and the conclusion.
- II. The lines containing the altitudes of a triangle are concurrent at the (*orthocenter*, *centroid*) of the triangle.
- III. The side opposite the vertex angle of an isosceles triangle is the (*hypotenuse*, *base*).
- IV. The linear equation $y = mx + b$ is in (*slope-intercept form*, *point-slope form*).

Multiple Choice

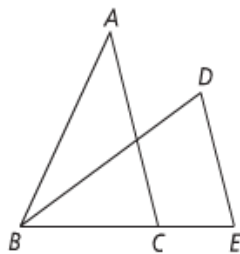
Read each question. Then write the letter of the correct answer on your paper.

1. One side of a triangle has length 6 in. and another side has length 3 in. Which is the greatest possible value for the length of the third side?
(A) 3 in. (C) 8 in.
(B) 6 in. (D) 9 in.
2. $\triangle ABC$ is an equilateral triangle. Which is NOT a true statement about $\triangle ABC$?
(F) All three sides have the same length.
(G) $\triangle ABC$ is isosceles.
(H) $\triangle ABC$ is equiangular.
(I) The measure of $\angle A$ is 50.

3. In the figure below, $\triangle ABC$ has vertices $A(-2, -4)$, $B(-3, 2)$, and $C(3, 0)$. D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{BC}$, and F is the midpoint of $\overline{AC}$. What are the coordinates of the vertices of $\triangle DEF$?



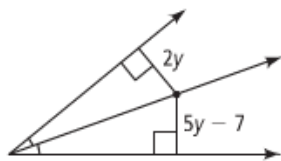
- (A) $D(-2.5, -1)$, $E(0, 1)$, $F(0.5, -2)$
 (B) $D(-2, -1)$, $E(0, 0.5)$, $F(0.5, -1)$
 (C) $D(-2, -1.5)$, $E(0, 1.5)$, $F(0.8, -2)$
 (D) $D(-2.2, -0.8)$, $E(0, 1.5)$, $F(0.5, -2)$
4. A square and a rectangle have the same area. The square has side length 8 in. The length of the rectangle is four times its width. What is the length of the rectangle?
- (F) 4 in. (H) 32 in.
 (G) 16 in. (I) 64 in.
5. In the figure below, $\angle A \cong \angle DBE$ and $\overline{AB} \cong \overline{BE}$. What additional information do you need in order to prove a pair of triangles congruent by AAS?



- (A) $\angle ABC \cong \angle BED$
 (B) C is the midpoint of $\overline{BE}$.
 (C) $\angle ACB \cong \angle BDE$
 (D) $\angle ABC \cong \angle BDE$

6. What is the value of y in the figure below?

- (F) $\frac{3}{7}$
 (G) 1
 (H) $\frac{7}{3}$
 (I) 3

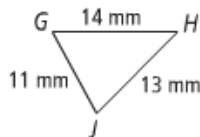


7. Which statement is the inverse of the following statement?

If $\overline{PQ}$ is a midsegment of $\triangle ABC$, then $\overline{PQ}$ is parallel to a side of $\triangle ABC$.

- (A) If $\overline{PQ}$ is a midsegment of $\triangle ABC$, then $\overline{PQ}$ is not parallel to a side of $\triangle ABC$.
 (B) If $\overline{PQ}$ is not a midsegment of $\triangle ABC$, then $\overline{PQ}$ is not parallel to a side of $\triangle ABC$.
 (C) If $\overline{PQ}$ is not parallel to a side of $\triangle ABC$, then $\overline{PQ}$ is not a midsegment of $\triangle ABC$.
 (D) If $\overline{PQ}$ is parallel to a side of $\triangle ABC$, then $\overline{PQ}$ is a midsegment of $\triangle ABC$.

8. Which statement is true for the figure below?



- (F) $m\angle J < m\angle G < m\angle H$
 (G) $m\angle H < m\angle G < m\angle J$
 (H) $m\angle J < m\angle H < m\angle G$
 (I) $m\angle H < m\angle J < m\angle G$

9. $\overline{AB}$ has endpoints $A(0, -4)$ and $B(8, -2)$. What is the slope-intercept form of the equation of the perpendicular bisector of $\overline{AB}$?

- (A) $y = -4x + 13$ (C) $y + 3 = -4(x - 4)$
 (B) $y = 4x - 19$ (D) $y + 3 = 4(x - 4)$

10. How can you prove that the two triangles at the right are congruent?



- (F) ASA (H) SAS
 (G) SSS (I) HL

11. Which statement contradicts the statement

$\triangle ABC \cong \triangle JMK$ by SAS?

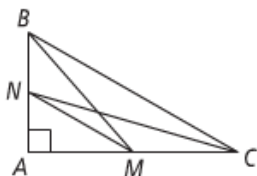
- (A) $\angle A$ and $\angle J$ are vertical angles.
 (B) $\overline{BC}$ is the hypotenuse of $\triangle ABC$.
 (C) $\triangle ABC$ is isosceles and $\triangle JMK$ is scalene.
 (D) $\overline{AB}$ is not congruent to $\overline{MK}$.

GRIDDED RESPONSE

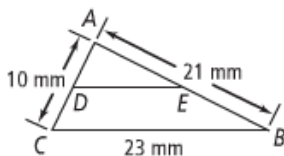
12. What is the next term in the pattern below?

5, 6, 8, 11, 15, ...

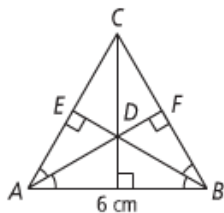
13. $\triangle ABC$ is a right triangle with area 14 in.^2 . $\overline{BM}$ and $\overline{CN}$ are medians and $BN = 2 \text{ in.}$ What is the area of $\triangle CNM$ in square inches?



14. The measure of one angle of an isosceles triangle is 70° . What is the measure of the largest angle?
 15. $\overline{DE}$ is a midsegment of $\triangle ABC$. In millimeters, what is DE ?



16. What is the area in square units of a rectangle with vertices $(-2, 5)$, $(3, 5)$, $(3, -1)$, and $(-2, -1)$?
 17. In the figure below, the area of $\triangle ADB$ is 4.5 cm^2 . In centimeters, what is DF ?



Short Response

18. If two lines are parallel, then they do not intersect. If two lines do not intersect, then they do not have any points in common.

- a. Suppose $\ell \parallel m$. What conclusion can you make about lines ℓ and m ?
 b. Explain how you arrived at your conclusion.

19. Copy the triangle below.



- a. Construct a congruent triangle.
 b. Describe your method.

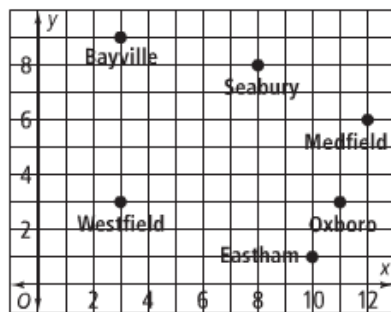
Extended Response

20. Write an indirect proof.

Given: $\triangle ABC$ is obtuse.

Prove: $\triangle ABC$ is not a right triangle.

21. The towns of Westfield, Bayville, and Oxboro need a cell phone tower. The strength of the signal from the tower should be the same for each of the three towns. The map below shows the location of each town, with each grid square representing 1 square mile.



- a. At what coordinates should the new cell phone tower be located? Explain.
 b. What is the distance from the new tower to Westfield? To Bayville? To Oxboro?
 c. Can any of the other towns shown on the map benefit from the new cell phone tower? Explain.